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Transportation Research Part B xxx (xxxx) xxx



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Ride-Sourcing modeling and pricing in non-equilibrium two-sided markets $\!\!\!\!^{\star}$

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ABSTRACT

Ride-sourcing is a prominent transportation mode because of its cost-effectiveness and convenience. It provides an on-demand mobility platform that acts as a two-sided market by matching riders with drivers. The conventional models of ride-sourcing systems are equilibrium-based, discrete, and suitable for strategic decisions. This steady-state approach is not suitable for operational decision-making where there is noticeable variation in the state of the system, denying the market enough time to balance back into equilibrium. We introduce a dynamic non-equilibrium ride-sourcing model that tracks the time-varying number of riders, vacant ride-sourcing vehicles, and occupied ride-sourcing vehicles. The drivers are modeled as earning-sensitive, independent contractor, and self-scheduling and the riders are considered price- and quality of service-sensitive such that the supply and demand of the ride-sourcing market are endogenously dependent on (i) the fare requested from the riders and the wage paid to the drivers and (ii) the rider's waiting time and driver's cruising time. The model enables investigating how the dynamic wage and fare set by the ride-sourcing service provider affect supply, demand, and states of the market such as average waiting and search time especially when drivers can freely choose their work shifts. Furthermore, we propose a controller based on the model predictive control approach to maximize the service provider's profit by controlling the fare requested from riders and the wage offered to drivers to satisfy a certain guality of market performance. We assess three pricing strategies where the fare and wage are (i) time-varying and unconstrained, (ii) time-varying and constrained so that the fare is higher than the wage such that the instantaneous profit is positive, and (iii) time-invariant and fixed. The proposed model and controller enable the ride-sourcing service provider to offer a wage to the drivers that is higher than the fare requested from the riders. The result demonstrates that this myopic loss can potentially lead to higher overall profit when customer demand (i.e., riders who may opt to use the ride-sourcing system) increases while the supply of ride-sourcing vehicles decreases simultaneously.

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M. Nourinejad and M. Ramezani/Transportation Research Part B xxx (xxxx) xxx

1. Introduction

Ride-sourcing is a popular mode of transportation because of its convenience and recent affordability caused by emerging mobile app platforms that match riders with drivers. The platform enables implementation of dynamic pricing schemes to improve the system's profit and level-of-service. Uber and Lyft, two major service providers, practice *surge-pricing*, which charges riders a higher fare during inclement weather conditions, major events, and rush hours. In cases of high demand, intricate pricing strategies balance the supply of drivers and the demand of riders. Uber claims that surge-pricing is favorable because it divides the riders into segments and serves them according to purchase power; riders that are rushed can pay the extra fare and others can wait until the price recedes (Uber, 2018). Surge-pricing can also induce a higher supply of drivers as they enter the market to collect a larger wage.

Ride-sourcing yields an interplay between drivers and riders in a two-sided market. The drivers find riders via the app platform. The ride-sourcing vehicle drives to the matched rider and then transports the rider to her designated destination. Upon dropping off the rider, the vehicle either becomes available again and cruises for the next rider or exits the market. The arrival of drivers and riders to the market depend on the pricing strategy, the fare collected from riders and the wage offered to drivers. The service provider can manipulate the fare and wage at each time step. The ultimate objective is to maximize the profit while ensuring a satisfactory level-of-service such as keeping the expected rider waiting time below a threshold. Different pricing strategies may achieve this goal: (1) *Static pricing* that is a scheme that keeps the fare and wage fixed throughout the planning period, (2) *Unconstrained dynamic pricing* that allows the fare and wage to vary without any constraints which may lead to solutions where the wage temporarily surpasses the fare, and (3) *Constrained dynamic pricing* that also varies the fare and wage but ensures that the latter is always lower than the former, thus generating a consistently positive profit throughout the planning period. The pricing strategy concurrently influences the arrival of riders and drivers to the system and their interaction in the two-sided market. In this paper, we present a dynamic model of ride-sourcing systems and investigate the impact of pricing (fare and wage) on the system's profit as a two-sided market.

The literature studies ride-sourcing pricing based on the steady-state equilibrium approach. Zha et al. (2017) model a ride-sourcing market using time-expanded networks to assess the impact of surge-pricing. The model is a bi-level program that sets prices at the upper-level to maximize revenue and finds the emerging equilibrium in the lower-level problem. In Zha et al. (2017), demand may be propagated from one time step to the next as long as the propagated demand remains constant in the time-space network. Cachon et al. (2017) study surge-pricing as a revenue management strategy. They consider a two-period newsvendor model to study the impact of contractual agreements for 'self-scheduling' agents, i.e., drivers that choose their own work-shifts in ride-sourcing markets. The study finds that surge-pricing is not necessarily optimal for profit maximization, but it achieves a near-optimal profit under fixed commission rates. Ma et al. (2018) develop a surge pricing method that is smooth in time and space where the ride-sourcing demand is exogenous and does not depend on the market state. Banerjee et al. (2015) use a queuing theory model to manipulate prices according to the number of riders in queue, and show that surge-pricing performs better when the platform has limited information. Bai et al. (2018) models ride-sourcing as an M/M/k queue in the steady-state condition and derives a static fare and wage to maximize the profit. The queuing model requires the ride-sourcing demand to be less than the supply to ensure stability. The steady-state static approach also leads to a wage always less than the fare. For future research, Bai et al. (2018) advocate for a more advanced dynamic model of the two-sided ride-sourcing market with optimal time-varying pricing strategies.

The equilibrium approach is prevalent in other studies of taxi markets for strategic decision-making such as taxi regulation (Yang et al., 2002; 2005a; Zha et al., 2016; Xu et al., 2017), demand analysis (Wong et al., 2001; 2008; Djavadian and Chow, 2017), taxi-shift modeling (Yang et al., 2005b; Zha et al., 2017), taxi-hailing (Wang et al., 2016; Amirgholy and Gonzales, 2016; Qian and Ukkusuri, 2017; Rahimi et al., 2018), search friction in taxi markets (Yang et al., 2010b; Yang and Yang, 2011), finding the optimal fleet size (Nourinejad and Roorda, 2014; 2016), bilateral matching (Najmi et al., 2017; Masoud and Jayakrishnan, 2017; Akbarpour et al., 2017), and the fare structure (Yang et al., 2010a; Qian et al., 2017). Yang et al. (2010b) and Wang et al. (2016) have shown the existence of equilibrium in taxi markets. However, they further argue that the equilibrium state is not unique and the requirements for a stationary equilibrium are strict and rarely prevailing in reality. Hence, for operational decision-making, equilibrium models may result in an inaccurate representation of the market when there are noticeable variations in the state of the system (i.e., number of riders and drivers) within a short time-frame (Hamedmoghadam et al., 2019). Under such abrupt changes, the system may not have enough time to balance itself back into equilibrium. Other drawbacks of using equilibrium to model dynamic ride-sourcing systems are that they (i) assume a constant time-invariant fleet size especially in taxi systems, (ii) use a discrete-hourly based modeling approach for strategic decision making, and (iii) do not consider the effect of temporal variations of fare and wage on entrance and exit rates of riders and drivers from the two-sided market. The discrete nature of the existing models with coarse intervals (e.g. hourly) makes them unresponsive to the highly time-varying changes in the system (Nie, 2017), and abrupt events (e.g., changes in weather conditions), which may substantially influence the supply of ride-sourcing vehicles and the demand of riders (Yang et al., 2005b; Zha et al., 2017; Xu et al., 2017). Furthermore, the equilibrium nature of the existing models limits their applicability in practice, especially when the system does not remain untouched for long enough to reach steady-state conditions. Thus, there is a need for non-equilibrium models that provide decision-making support at the operational level.

Empirical evidence shows that ride-sourcing markets are subject to search friction as drivers cruise to the riders to serve. A prominent approach to model search friction in ride-sourcing systems is through bilateral meeting functions (Yang et al., 2010b; Yang and Yang, 2011). The meeting function was first introduced in labor markets (Mortensen and Pissarides, 1994;

3

M. Nourinejad and M. Ramezani/Transportation Research Part B xxx (xxxx) xxx

Petrongolo and Pissarides, 2001), as a means of characterizing the rate that two agents meet each other, e.g., the rate that employers meet job candidates. Schroeter (1983), Lagos (2000), and Yang and Yang (2011) showed that ride-sourcing markets exhibit the same type of friction where vacant vehicles search for riders. The friction depends on the state of the system defined as the number of riders and vacant vehicles. Ramezani and Nourinejad (2017) further demonstrated and quantified the effect of network congestion (network average speed) on the driver-rider search friction. This enhanced model is used in Ramezani and Nourinejad (2017) to devise an optimal taxi dispatch controller that captures the interrelated impact of normal traffic flows and taxi dynamics using the concept of the macroscopic fundamental diagram. In this paper, we use dynamic bilateral meeting functions to introduce an analytical ride-sourcing model where fare and wage, explicitly set by the service provider, impact the supply and demand of the market and the profit of the system. The supply and demand are endogenously derived and sensitive to the fare requested from riders and the wage offered to drivers.

This paper presents two models: (i) A macroscopic, analytical, and parsimonious three-state model (riders, vacant ridesourcing vehicles, and occupied ride-sourcing vehicles) where the arrivals of vehicles and riders are endogenously dependent on the market state, and (ii) a microscopic discrete-event agent-based simulation model that considers individual riders and ride-sourcing vehicles. The microsimulation model is detailed and disaggregated as it tracts the position and status of each rider and driver over the planning horizon. The status of the vehicles is either vacant or occupied and the status of riders is either waiting or picked-up. The microsimulation model considers all the interactions of the ride-sourcing market at the level of individual agents, i.e., vehicles and riders. Every rider has her own characteristics including request time, pick up time, drop off time, origin, and destination. Each vehicle joins and exits the market based on her perception of the instantaneous utility of the market that is a function of wage and search time. The vehicle serves the rider assigned to it by the platform by driving to the location of the rider, and taking the rider to her destination. Then, the vacant vehicles search for the next rider or exit the system. This decision is again based on the driver's perception of the instantaneous utility of the market. The vehicles are occupied while they carry riders between the origin and the destination. The microsimulation model is used to verify the aggregate modeling assumptions that are needed to derive the analytical model and it serves as a proxy of reality to test the effect of optimal dynamic fare and wage.

In contrast, the macroscopic three-state model is developed to offer a parsimonious representation of the ride-sourcing market capturing intricate interactions between the time-varying demand and supply with realistic assumptions on the aggregated dynamics of the market. This model is further used to control the ride-sourcing pricing, i.e., fare and wage. The model may result in situations where the wage of ride-sourcing vehicles is greater or equal to the fare of ride-sourcing customers, i.e. a short period of time with negative profit, such that the overall profit of the ride-sourcing platform over the control time horizon is maximized. Note that no equilibrium-based static model can lead to this result as in a static model (i.e., time-invariant fare and wage) where the fare always exceeds the wage the profit is strictly positive. This is a crucial methodological contribution as the majority of the ride-sourcing literature focuses on equilibrium modeling of the market.

The proposed models enable a dynamic regulatory reform to benefit the ride-sourcing riders, drivers, and platform simultaneously. We develop a dynamic optimal pricing method based on the Model Predictive Control (MPC) approach that maximizes the service provider's profit while offering an acceptable level-of-service by keeping the wait times low. The MPC approach is successfully applied in other transportation systems, for instance in (Geroliminis et al., 2013) and (Ramezani et al., 2015); it is also suitable for controlling the ride-sourcing system because (i) it allows us to foresee the impact of the chosen prices on the future states of the system and (ii) it offers tractable and real-time control solutions for systems with state and control constraints. A methodological contribution of this paper is to use two inherently different models in the MPC framework as the plant (reality) and as the optimization model. For the plant, we use the microsimulation model to represent a detailed representation of ride-sourcing markets where the arrival and departure of agents (i.e., riders and vacant vehicles) are functions of time-varying fare, wage, waiting time, and search time. For the optimization model, we use the macroscopic analytical model based on the dynamic meeting function that models the number of riders, vacant vehicles, and occupied vehicles. We assess the impact of three pricing strategies and show that the unconstrained dynamic pricing method has a profit advantage of 11% and 32% over the constrained dynamic pricing method and the static pricing method, respectively. The superior pricing strategy also results in a lower rider waiting time and driver cruising time in the system.

The remainder of this paper is as follows. In Section 2, we develop the analytical and macroscopic model of the two-sided ride-sourcing market by modeling the dynamics of riders and vacant and occupied ride-sourcing vehicles. In Section 3, we introduce the simulation-based and microscopic model of the ride-sourcing market and investigate the modeling assumptions that are needed to derive the macroscopic analytical model such as the search friction considering different matching mechanisms. In Section 4, we present the optimal fare and wage control problem and propose a controller based on the MPC approach. In Section 5, we assess the model and the controller using numerical experiments and compare the three pricing strategies. In Section 6, we conclude the study and outline the future research directions.

2. Macroscopic modeling of two-sided ride-sourcing markets

Consider a region that holds p(t) waiting riders and v(t) vacant vehicles at time t. We denote the boarding rate (used interchangeably with the meeting rate) with m(t) = M(p(t), v(t)) so that m(t) riders board vacant vehicles between t and t + dt. Throughout this paper, it is assumed that the requested service of any rider can be met by any of the vacant ride-sourcing vehicles. The meeting rate increases with v(t) and p(t) such that $\frac{\partial M(t)}{\partial v(t)} > 0$ and $\frac{\partial M(t)}{\partial p(t)} > 0$, $\forall t$. Moreover, $m(t) \rightarrow 0$

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M. Nourinejad and M. Ramezani/Transportation Research Part B xxx (xxxx) xxx

when either $v(t) \rightarrow 0$ or $p(t) \rightarrow 0$, i.e., there are no boardings (meetings) in the absence of riders or vacant vehicles. Intuitively, $M(p(t), v(t)) \le p(t)$ and $M(p(t), v(t)) \le v(t)$.

The literature of ride-sourcing meeting functions is limited to steady-state conditions where p(t) and v(t) are at equilibrium and time-independent, i.e., p(t) = p and v(t) = v. Nevertheless, we demonstrate that the bilateral meeting function can be extended to time-varying conditions. The time-varying macroscopic bilateral meeting function can be defined using the following two time-independent elasticities γ_1 and γ_2 :

$$\gamma_{1} = \frac{\partial M(t)}{\partial v(t)} \frac{v(t)}{M(t)} \quad \forall t$$

$$\gamma_{2} = \frac{\partial M(t)}{\partial p(t)} \frac{p(t)}{M(t)} \quad \forall t,$$
(2)

where $0 < \gamma_1$, $\gamma_2 \le 1$. A common functional form for the meeting function, satisfying all the above conditions, is the following Cobb-Douglas function:

$$M(p(t), v(t)) = a_0 \left(v(t) \right)^{\gamma_1} \left(p(t) \right)^{\gamma_2}, \tag{3}$$

where a_0 is a fixed positive constant. In Section 3, we quantitatively investigate this modeling formulation considering different micro-matching mechanisms between drivers and riders.

Consider a single ride-sourcing firm in a deregulated market that is able to control its fare and wage in real-time. The state of the system is defined by the triplet (p(t), v(t), o(t)) where p(t) is the number of riders waiting to be picked up, v(t) is the number of vacant ride-sourcing vehicles, and o(t) is the number of occupied ride-sourcing vehicles at time t. The number of riders changes dynamically based on the arrival rate of riders (i.e., the demand which can be assumed to be a function of the state of the ride-sourcing system such as fare, rider waiting time, etc.) and the meeting rate M(t). Hence,

$$\frac{\mathrm{d}p(t)}{\mathrm{d}t} = A_{\mathrm{p}}(t) - M\Big(p(t), v(t)\Big),\tag{4}$$

where $A_p(t)$ is the arrival rate of waiting riders which strictly decreases with the expected cost of riders at time *t*. The generalized cost of riders is defined as $C_p(t) = f(t) + \alpha_p w_p(t)$ where f(t) is the fare at time *t*, $w_p(t)$ is the expected waiting time of riders who enter the market at time *t*, and α_p is the marginal cost of waiting time which is positive. The fare that the operator charges riders, f(t), is a control measure set in real-time to optimize a designated objective function (e.g., maximizing the service provider's profit). The expected waiting time of riders who enter the ride-sourcing market at time *t* is modelled as:

$$w_{\rm p}(t) \approx \frac{p(t)}{M(t)}.$$
 (5)

Note that the RHS of Eq. (5) is the expected waiting time if the meeting rate and the number of riders remain constant over a sufficiently long period. Essentially, Eq. (5) is Little's formula that holds true at the steady state conditions. In dynamic cases, Eq. (5) is an approximation, i.e., the ratio between instantaneous number of riders and the number of meetings can be used to estimate (with certain degree of inaccuracy) the expected waiting time of riders. The accuracy of this approximation is investigated in Section 3; see Fig. 4.

Let $D_p(t)$ [veh/unit of time] be the exogenous travel demand at time *t* constituting all travel modes where the ridesourcing is a travel choice among the set of travel modes. The rider arrival rate $A_p(t)$ is a proportion of $D_p(t)$ which is denoted as $U_p(C_p(t))$ such that

$$A_{\rm p}(t) = D_{\rm p}(t) \cdot U_{\rm p}\Big(C_{\rm p}(t)\Big),\tag{6}$$

where $0 \le U_p \le 1$, because the rider arrival rate into the ride-sourcing market is only a part of the total travel demand. Furthermore, $\frac{\partial U_p}{\partial c_p} \le 0$, because the arrival rate of riders to the ride-sourcing market decreases with the expected cost. The travel demand $D_p(t)$ is assumed to be independent of ride-sourcing dynamics yet time-varying, while the arrival rate $A_p(t)$ is affected by the real-time fare and service quality. Note that a more sophisticated discrete choice model can be used instead of Eq. (6), however, identifying and estimating the utility of other available modes is more data-intensive and is not within the scope of this model.

Occupied vehicles become vacant when they drop off their rider. The turnover rate from occupied to vacant is G(o(t)) where $\frac{\partial G}{\partial o} > 0$, $\forall t$; the turnover rate increases with the number of occupied vehicles which is evident. G(o(t)) denotes the trip completion rate of occupied vehicles that is a function of the number of occupied vehicles, trip length of riders, and congestion level of the network. The following equation models the evolution of vacant ride-sourcing vehicles:

$$\frac{d\nu(t)}{dt} = A_{v}(t) - M\left(p(t), \nu(t)\right) + G(o(t)) - E_{v}(t),$$
(7)

where $A_v(t)$ is the arrival rate and $E_v(t)$ is the exit rate of vacant ride-sourcing vehicles to and from the ride-sourcing market at time *t*, respectively.

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4

M. Nourinejad and M. Ramezani/Transportation Research Part B xxx (xxxx) xxx

5

To model the arrival and exit rates of vacant ride-sourcing vehicles, we define a proxy of market benefit attractiveness from the perspective of drivers, $C_v(t)$. Intuitively, the higher the market benefit attractiveness the higher is the arrival rate of ride-sourcing vehicles to the market, i.e. $\frac{\partial A_v(t)}{\partial C_v(t)} \ge 0$, and the lower is the exit rate of the ride-sourcing vehicles from the market, i.e. $\frac{\partial E_v(t)}{\partial C_v(t)} \le 0$. The proxy of market benefit attractiveness to drivers at time t, $C_v(t)$, includes (i) current wage offered to the drivers by the platform at time t, g(t), (ii) current expected cruising time of vacant ride-sourcing vehicles starting their cruising at time t, $w_v(t)$, (iii) cost incurred by occupied rides, and (iv) expected prediction of (dis)benefit of the market forecast at time t, $\hat{C}_v(t)$. The first two terms are endogenous; g(t) is the control variable and $w_v(t)$ is a dynamic state of the market. The third term, cost incurred by occupied rides, can be modelled as the product of the average occupied-trip travel time, l, and the time-value of drivers, β_v , measured in \$/hr. Note that β_v includes fuel cost, vehicle depreciation, and all other indirect costs. Then, the occupied travel cost per ride is $\beta_v l$, which is similar to the models of the literature, e.g., Yang and Yang (2011). Since l is a exogenous parameter, in this paper, l and β_v are considered as constant parameters for simplicity, and accordingly, the cost incurred by occupied rides is considered as a fixed term in the proxy of market benefit attractiveness that may be dropped for brevity.

The fourth term is the expected prediction of (dis)benefit of the market forecast at time t, $\hat{C}_{y}(t)$, which represents the driver perception of the future benefit of the market. This includes future waiting time and occupied time that is a function of future demand, supply, fare, wage, etc. Though it is a crucial variable, modeling such prediction as an aggregated dynamic model is challenging and complex. One reason is that the ride-sourcing market is a decentralized economy where future states of the market depend on the future actions of all the potential drivers. In addition, considering the dynamic nonequilibrium nature of our model, it is impractical to assume that drivers have perfect information of the market states in future, especially when the market changes with unexpected peaks/drops in demand, supply, fare, and wage. Consequently, we consider only the instantaneous proxy of market benefit attractiveness to drivers. That is, we assume drivers have a null prediction of future states of the market and make the determination to enter or exit the market on the basis of current information of the market, that is $C_V(t) = g(t) - \alpha_V w_V(t)$. Hence, $C_V(t)$ is the proxy of market benefit attractiveness from the perspective of drivers or generalized benefit of vehicles at time t, $w_v(t)$ is the expected cruising time of vacant vehicles starting their cruising at time t, and $\alpha_{\rm V} > 0$ is the marginal cost of cruising time. The wage that the platform pays to the drivers, g(t), is a control measure and can be manipulated in real-time to optimize the designated objective function. Note that still with the proposed simplifications, the model is detailed enough to capture realistic dynamics of the market such that a dynamic fare and wage controller can be developed for the market. It is worth mentioning that the overall controller design can be used with any other sensible aggregated supply model of vacant vehicles.

The cruising time of vacant vehicles to find a rider starting their cruising at time t is modelled as:

$$w_{\rm v}(t) \approx rac{v(t)}{M(t)}.$$
 (8)

Similar to Eq. (5), the RHS of Eq. (8) is the expected cruising time of a ride-sourcing vehicle that starts its cruising to reach to a rider at time *t* if the meeting rate and the number of vacant ride-sourcing vehicles remain constant over a sufficiently long period. Eq. (8) is essentially Little's formula that holds true at the steady state conditions, while in dynamic cases, Eq. (5) is an approximation. The accuracy of this assumption is investigated in Section 3, see Fig. 4.

Let $S_v(t)$ [veh/unit of time] be the maximum supply of drivers that are willing to enter the market at time *t*. The driver arrival rate $A_v(t)$ is a proportion $U_v^{in}(C_v(t))$ of the supply $S_v(t)$ of drivers that enter the system at time *t* such that

$$A_{\rm v}(t) = S_{\rm v}(t) \cdot U_{\rm v}^{\rm in} \Big(C_{\rm v}(t) \Big), \tag{9}$$

where $0 \le U_v^{\text{in}} \le 1$, and $\frac{\partial U_v^{\text{in}}}{\partial C_v} \ge 0$ because more drivers enter the system as the benefit $C_v(t)$ increases. In a similar way, we model the vacant vehicle exit rate, $E_v(t)$, as a proportion $U_v^{\text{out}}(C_v(t))$ of the number of the vacant vehicles in the market, v(t), that decide to leave the market at time t such that

$$E_{\mathbf{v}}(t) = \mathbf{v}(t) \cdot U_{\mathbf{v}}^{\text{out}}\Big(C_{\mathbf{v}}(t)\Big),\tag{10}$$

where $0 \le U_v^{\text{out}} \le 1$, and $\frac{\partial U_v^{\text{out}}}{\partial C_v} \le 0$ because more drivers leave as the benefit $C_v(t)$ decreases. Note that with $U_v^{\text{out}} = 0$, all existing drivers decide to stay in the market, and with $U_v^{\text{out}} = 1$ all vacant vehicles exit the market. The arrival and departure rate of vehicles in the market is a variable that the service provider can indirectly manipulate by controlling the wage. This is further explored in Sections 4 and 5.

There are two primary theories on how ride-sourcing drivers choose their work-shift hours. The neoclassical theory claims that the entry and exit rates depend on the wage chosen by the service provider. In contrast, the income-targeting approach allows drivers to exit the market when they reach an earnings threshold for that shift. There is little empirical consensus on whether one theory outperforms the other (Farber, 2015). Zha et al. (2017) formulate both strategies as an equilibrium model and investigate their properties. In our proposed model, the entry and exit strategies of ride-sourcing drivers are similar to the neoclassical approach where the drivers are triggered to enter or exit the market based on the instantaneous utility defined as the generalized benefit. That is, it is assumed that the drivers have a null prediction of market states in short-term future.

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M. Nourinejad and M. Ramezani/Transportation Research Part B xxx (xxxx) xxx

Finally, Eq. (11) models the dynamics of the number of occupied ride-sourcing vehicles:

$$\frac{\mathrm{d}o(t)}{\mathrm{d}t} = M\left(p(t), v(t)\right) - G(o(t)),\tag{11}$$

where the number of occupied vehicles increases with the meeting rate, M(t), and decreases with the trip completion rate of occupied vehicles, G(o(t)). Note that the trip completion rate of occupied vehicles is a function of the number of occupied vehicles, trip length of occupied vehicles, and congestion level of the network (Ramezani and Nourinejad, 2017). Without loss of generality, in the numerical tests of this paper, we assume G(o(t)) has a linear relationship with o(t). A more comprehensive functional form of trip completion rate of occupied vehicles (e.g. see Ramezani and Nourinejad, 2017) can be readily considered in the numerical experiments.

3. Micro modeling of two-sided ride-sourcing markets

In this section, we introduce a dynamic micro-simulation model of ride-sourcing systems. The micro-simulation model is event-based where each non-identical rider has an origin, destination, and a request time. The riders wait at their origin until they are picked up by a driver. The experienced waiting time of each rider is the time lapse between the request time and the pick up time. The rider's decision of entering the system depends on the fare and the expected waiting time at the time of request. The ride-sourcing vehicles are either vacant and searching for rider or they are occupied and en-route to the on-board passenger's destination. The vacant vehicles experience a cruising time which is the time lapse between the last drop off and the next pick up. The drivers can choose to exit the system at any time of their choosing during the cruising state. The driver's decision of entering and leaving the system depends on the wage and the expected cruising time at the time of the decision. This model allows to capture heterogeneity in rider trip details, i.e. origin, destination, and request time, as well as heterogeneity in the location and time at which the ride-sourcing vehicles enter and exit the market.

The simulation model allows us to estimate the meeting rate m(t) between the rise-sourcing riders and vehicles w.r.t. the number of waiting riders p(t) and vacant vehicles v(t) at time t. Furthermore, the microsimulation model allows to validate the proposed macroscopic meeting function in Eq. (3). To this end, we observe the number of boardings of riders by the ride-sourcing vehicles in the microsimulation model and compare with the analytical macroscopic meeting function. To estimate the meeting function, we proceed with a grid network with bi-directional streets. The ride-sourcing vehicles travel a full-length street at each time step, i.e., each vehicle travels from one intersection to the next. Vehicle boarding occurs when a vacant vehicle meets a rider on a street. We consider two matching methods (or search strategies), (i) Random cruising and (ii) Greedy dispatch. The random cruising method replicates the search dynamics of traditional taxis where no information of the location of riders is available. Hence, vehicles search randomly; when a vehicle reaches an intersection, it chooses one of the three available directions with the same probability (without loss of generality, U-turns are not allowed). They continue searching until they find a rider. The greedy dispatch method replicates recent app-based platforms where riders and vacant vehicles are matched through a central system. We assign each newly arrived rider to the nearest vacant vehicle. Assigned vacant vehicles change their searching strategy and head to their designated rider using the shortest path. In case another rider is along the shortest path, the vehicle ignores it and continues to pick up the designated rider. The framework of this modeling is depicted in Fig. 1. The average meeting rates are depicted in Figs. 2 and 3 for the greedy dispatch and random cruising search methods, respectively. Fig. 2(a) shows the average meeting rate in the micro model for a range of riders and drivers. Note that the number of drivers and riders in the discrete event simulation model are not constant and



Fig. 1. The schematic framework of the event-based microsimulation model of the ride-sourcing system.

M. Nourinejad and M. Ramezani/Transportation Research Part B xxx (xxxx) xxx



Fig. 2. (a) Meeting rate from the simulation model with the greedy dispatch search method of vacant vehicles. (b) Meeting rate based on the estimated meeting function in Eq. (3).



Fig. 3. (a) Meeting rate from the simulation model with random cruising search method of vacant vehicles. (b) Meeting rate based on the estimated meeting function in Eq. 3.

change dynamically. Fig. 2(b) displays the estimated meeting rate, i.e., Eq. (3) with best fit based on error minimization with respect to the synthetic data from the microsimulation model. Fig. 3(a) depicts the average meeting rate in the simulation model while Fig. 3(b) displays the estimated meeting rate in Eq. (3) for the random cruising search method. For the greedy dispatch search method, the meeting rate function is estimated as $M(p(t), v(t)) = 0.06v(t)^{0.68}p(t)^{0.81}$ where $R^2 = 0.95$ and the two estimated elasticities are statistically significant. For the random cruising method, the meeting rate function is estimated as $M(p(t), v(t)) = 0.005v(t)^{0.98}p(t)^{0.47}$ where $R^2 = 0.97$ and the two estimated elasticities are statistically significant. For the random cruising method, the meeting rate function is estimated as $M(p(t), v(t)) = 0.005v(t)^{0.98}p(t)^{0.47}$ where $R^2 = 0.97$ and the two estimated elasticities are statistically significant. An alternative approach is to estimate a unique value of a_0 for both matching methods.¹ Accordingly, for the greedy dispatch method, we obtain $M(p(t), v(t)) = 0.015v(t)^{0.83}p(t)^{0.98}p(t)^{0.98}$ with $R^2 = 0.91$ and for the random cruising search method, we obtain $M(p(t), v(t)) = 0.015v(t)^{0.84}p(t)^{0.41}$ with $R^2 = 0.91$. When estimating the meeting functions, we find the optimal unique value of $a_0 = 0.015$ for both random cruising and greedy dispatch methods.

Comparison of the two search methods has the following insights. First, we observe that the bilateral meeting function models ride-sourcing markets with a high level of accuracy regardless of the implemented micro-level matching mechanism; the R^2 of the two models is above 0.91 and the parameters are statistically significant. This allows us to use calibrated

¹ In one view, a_0 can represent the effect of the size of the network, spatial characteristics (e.g. heterogeneity level) of demand and supply of the ridesourcing market, and the matching method (e.g. greedy dispatch or random cruising). That is, the effect of the matching method is partially captured in a_0 and elasticities. Alternatively, one can assume a_0 does not depend on the matching method and only elasticities, γ_1 and γ_2 , capture the effect of the matching method. Note that capturing the effect of matching method in a_0 parameter is a rather philosophical concern that might be a source of different views on interpretation of the meeting function and its parameters including elasticities and the value of returns to scale. This needs further investigations of real-world data.

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M. Nourinejad and M. Ramezani/Transportation Research Part B xxx (xxxx) xxx



Fig. 4. (a) Simulated rider waiting time compared to the analytical waiting time from Eq. (5). (b) Simulated driver cruising time compared to the analytical cruising time from Eq. (8).

bilateral meeting functions to find optimal pricing strategies that maximize profit. Second, we observe that the greedy dispatch and random cruising matching methods have increasing returns to scale as $\gamma_1 + \gamma_2 > 1^2$ (Assuming unique value of a_0 for estimation of the meeting rate function, the superior efficiency of the greedy dispatch model with respect to random cruising is portrayed in a greater value of returns to scale. Note that the observed meeting rates of the two search methods differ by an order of magnitude.) Third, the asymmetry of the meeting function defined as γ_1/γ_2 in the random cruising method is grater than 1. See (Yang et al., 2014) where a similar observation is reported based on empirical findings. In contrast, in the greedy dispatch method, the asymmetry is less than 1. See Schroeter (1983) where a similar observation is reported based on empirical findings. Another interpretation is that the greedy dispatch model has $\gamma_1 < \gamma_2$, thus, addition of a rider to the system has a higher impact on the meeting rate than adding a vacant vehicle. However, the opposite occurs in the random cruising mechanism where vacant vehicles have a higher impact on the meeting rate, $\gamma_2 < \gamma_1$, because vacant vehicles are moving agents searching for stationary riders.

The investigation of both the greedy dispatch and random cruising matching strategies is to demonstrate that the proposed dynamic Cobb-Douglas model of the meeting rate in Eq. (3) can capture different types of matching mechanisms between ride-sourcing vehicles and riders. In other words, while the greedy dispatch and random cruising strategies are matching mechanisms at the micro and agent level, the aggregated dynamics of the number of boardings at the market level can well be reproduces by the parsimonious Cobb-Douglas type model proposed in Eq. (3).

The total waiting time in e-hailing (greedy dispatching) consists of the matching time (the time needed for the e-hailing platform to match the request with a vacant ride-sourcing vehicle) and the time needed for the driver to physically reach the rider to pick her up. We assume that the matching time is zero which holds true as long as there is always one vacant vehicle in the e-hailing platform. In contrast, rider waiting time in on-street searching (random cruising) is the time a rider has to wait until a driver finds her.

We use the microsimulation model to verify the validity and accuracy of the rider waiting time, Eq. (5), and ride-sourcing vehicle cruising time, Eq. (8). Upon boarding of each rider by a ride-sourcing vehicle, we measure her waiting time from the simulation model, the instantaneous number of the waiting riders, p(t), and the instantaneous meeting rate, M(t), estimated as in Eq. (3). Then the aggregate model of rider waiting time is estimated as Eq. (5). Each observation is presented as a data point in Fig. 4(a). The same procedure is carried out for driver cruising time in Fig. 4(b). We show that the analytical model is reasonably accurate in estimating the rider waiting time and the driver cruising time. The R^2 values are 0.89 and 0.84 for the waiting time, respectively. The mean absolute error is 0.65 and 0.82 [min] for the waiting time and cruising time, respectively. The mean absolute error is inherently a discrete model both in time and in unit of riders and drivers, whereas the analytical macroscopic model presented in Section 2 is a continuous model. Accordingly, the microsimulation model serves as the plant in the MPC framework while we embed the dynamics of the

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² We conjecture that the returns to scale depend on the matching method of the service provider and the embedded technology in the matching mechanism. We cannot make a general assertion about the relative order of returns to scale of markets with dispatching (or cruising) because there are many un-modelled factors involved in the ride-sourcing market, including matching technology, level of traffic congestion, existence of taxi stands, and level of demand and supply, among others. Further studies of real-world data are required to shed more light on the general properties of returns to scale of ride-sourcing markets with different matching methods.

q

(12b)

ride-sourcing market presented in (3)-(11) in the MPC framework as the optimization model. The details of the dynamic pricing of the ride-sourcing market and the MPC framework are presented in the next section.

4. Optimal dynamic pricing of two-sided ride-sourcing market

4.1. Constrained optimization formulation

We consider the objective of the ride-sourcing provider is to maximize the overall profit subject to reasonable level of service constraints. The ride-sourcing provider can control the service pricing that is the fare, f(t), and wage, g(t), in real-time to maximize the total profit of the system over a period of time between t_0 and t_f . The total profit is the integral of point difference between the fare and wage weighted by the number of meetings within the desired time interval. The formulation of the optimal dynamic pricing of the ride-sourcing market subject to the dynamics of the market as modelled in (3)–(11) and quality of service of the market is as follows:

$$\max_{f(t),g(t)} \qquad J = \int_{t_0}^{t_f} \left[A_p(t) \cdot f(t) - m(t) \cdot g(t) \right] dt,$$
(12a)

subject to (3) - (11)

$$w_{\rm p}(t) \le W_{\rm p}^{\rm max} \qquad t \in [t_0, t_{\rm f}]$$
(12c)

$$w_{v}(t) \leq W_{v}^{\max} \qquad t \in [t_{0}, t_{f}]$$
(12d)

$$f(t), g(t) \ge 0 \qquad \qquad t \in [t_0, t_f]$$
(12e)

$$v(t), p(t), o(t) \ge 0$$
 $t \in [t_0, t_f]$ (12f)

The optimization (12) is a constrained, nonlinear, and non-convex problem. The first term in (12a), $A_p(t) \cdot f(t)$, is the revenue from riders that pay the fare at their arrival time to the system. The second term in the objective function, $m(t) \cdot g(t)$, is the total wage paid to the drivers at the time they are matched with a rider. This pricing arrangement has a close resemblance to the current state of practice in prime ride-sourcing companies. Uber and Lyft calculate a fare for each rider according to her trip characteristics and the state of the system. The rider can choose to accept or reject the ride without committing to the requested fare. Constraints (12c) and (12d) ensure that the vehicles and riders experience a satisfactory level of service by keeping the instantaneous rider wait time, $w_p(t)$, and vehicle cruising time, $w_v(t)$, below a predefined threshold of W_p^{max} and W_v^{max} , respectively. Constraint (12e) is a control action constraint that ensures non-negativity of the fare and wage. Similar constraints can be imposed to enforce different bounds on the fare and wage. For example, one may cap the maximum fare to eliminate unrealistically high prices. One may also satisfy government mandated minimum wage conditions under additional constraints. Constraint (12f) guarantees that the state of the model remain non-negative. Note that optimization problem (12) comprises the macro-state dynamics of the rides-sourcing market as presented in (3)–(11). To solve (12), it is assumed that the supply of drivers, $S_v(t)$, travel demand, $D_p(t)$, and initial value of the states, $p(t_0)$, $v(t_0)$, and $o(t_0)$ are given.

We consider the following three pricing strategies that requires additional constraints to be added to the optimization problem (12) to enforce the intended pricing strategy.

- 1. *Static pricing*: This strategy assumes a fixed fare and wage throughout the planning period. This strategy resembles the solution of equilibrium-based modeling approach of the ride-sourcing market. Accordingly, the two time-invariant decision variables are f(t) = f and g(t) = g, for $t \in [t_0, t_f]$.
- 2. Constrained dynamic pricing (CDP): This strategy assumes that the instantaneous profit is always non-zero by setting the fare to be always larger than the wage by an offset Δ . To implement CDP strategy, it is required to augment the following constraint to optimization problem (12):

$$f(t) - g(t) \ge \Delta, t \in [t_0, t_f].$$

(13)

3. Unconstrained dynamic pricing (UDP): This strategy does not impose any restrictions on the relative order of price or wage, thus allowing the instantaneous profit to be negative if necessary.

4.2. Model predictive control for optimal pricing

We solve the optimal fare and wage control problem (12) using a receding horizon approach implemented in the model predictive control (MPC) framework. The MPC offers tractable and real-time solutions for dynamic systems with state and control constraints (Sirmatel and Geroliminis, 2017). The MPC is a control approach that is based on a prediction model to predict the future states of the system and to optimize the control actions accordingly based on the current and predicted



M. Nourinejad and M. Ramezani/Transportation Research Part B xxx (xxxx) xxx





Fig. 5. Model predictive control (MPC) scheme for ride-sourcing pricing. Note the systematic difference between the plant which is a discrete-event microsimulation and the prediction model which is based on the macroscopic analytical model of riders and vacant and occupied vehicles.

future states. (In this paper, 'optimization model' and 'prediction model' are used interchangeably.) The MPC predicts the state of the system using the prediction model over a horizon period of N_p time steps into the future considering a sequence of optimal control inputs with a length of N_c time steps. The MPC obtains the control inputs by solving a multi-state, finite-horizon, constrained, and open-loop optimal control problem at each time step. This finite-horizon optimal control problem takes into the account the predicted states of the system which are forecast based on the information feedback from the plant that provides the initial state of the prediction model, see Fig. 5. The MPC then takes the first time step of the control actions and implements it on the plant, i.e. in this paper the microsimulation model presented in Section 3. The procedure is then repeated with a shifted rolling horizon until the entire control period, i.e. $t \in [t_0, t_f]$, is covered. This framework is conceptually presented in Fig. 5. In the bottom block, the *price controller* takes the current state of the prediction model fare and wage to maximize the profit of the prediction model presented in Section 2 over the prediction horizon of N_p time steps. The output of the *price controller*, the optimal fare and wage, is passed to the *plant* as shown at the top block of Fig. 5. The interaction between the *plant* and the *price controller* continues throughout the entire control period.

In this paper, we adopt the direct sequential method for solving the optimization problem where its core principle is to first discretize the problem and then find the optimal control actions. As a result, this method can use state-of-theart nonlinear optimization solvers (e.g. interior-point method). The proposed MPC optimizes a discrete reformulation of Eq. (12a) over N_p time steps by employing an iterative nonlinear optimization solver. The discretization is as follows. The pricing actions, f(t) and g(t), without loss of generality, are assumed to change every K_c step. (The duration of time step K_c is set to 5 [min] in the numerical tests.) The prediction model is temporally discretized as the approximate solution of the set of ordinary differential Eqs. (3)–(11) with a step size significantly smaller than K_c . Hence, within each K_c step of the prediction model, the fare and wage remain constant, however, the states that are the number of riders, vacant vehicles, and occupied vehicles are time-varying.

Note that the selection of prediction horizon N_p and the control horizon N_c affects the performance of the MPC approach. The prediction horizon should be long enough to enable the prediction model to forecast the states accurately. The trade-off rises as a large N_p increases the optimization computation complexity, which may hinder real-time implementation. Similar

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JID: TRB

M. Nourinejad and M. Ramezani/Transportation Research Part B xxx (xxxx) xxx

11



Fig. 6. Supply (drivers) and demand (riders) of the market during the peak control period. Not all of the supply and demand join the ride-sourcing market. The actual arrival rate of riders depends on the fare that the ride-sourcing firm charges and the instantaneous waiting time. The actual arrival rate of vacant vehicles is a function of wage they receive from the ride-sourcing firm and the instantaneous cruising time.

attention is required for the control horizon N_c to balance the trade-off between computation resources and optimality of the results. Note that $N_c \leq N_p$ to reduce the real-time computational complexity.

5. Numerical experiments

The case study considers a single ride-sourcing firm operating in a metropolitan area made of a grid network of 10×10 bi-directional streets. The rider demand, $D_{\rm p}(t)$, represents a time-varying peak period. Note that $D_{\rm p}(t)$ is the total rider demand with only a proportion using the ride-sourcing mode based on the fare and rider waiting time. In other words, the total travel demand is the same for different pricing strategies, however, the number of ride-sourcing riders depends on the pricing strategy and the market characteristics. In this case study, the vehicle supply, $S_{\rm V}(t)$, has a negative peak as we assume the drivers are mainly joining the market as casual employees who have a different main occupation. Note that $S_{\rm V}(t)$ is the potential supply where only a proportion join the ride-souring market as the vacant vehicles. Fig. 6 illustrates $S_{\rm v}(t)$ and $D_{\rm p}(t)$ for four different cases. Case I represents a jump in demand and drop in supply (this considers other employment opportunities for drivers outside the ride-sourcing market); Case II represents a similar pattern to Case I with a larger maximum demand and lower minimum supply; Cases III and IV are also similar to Case I but have different timing in the extreme points of the supply and demand. There are initially 100 riders and 200 vehicles in the system, i.e., p(0) = 100, v(0) = 200, and o(0) = 0. The control final time, $t_{\rm f}$, is set to 180 [min], the maximum allowable waiting time, $W_{\rm p}^{\rm max}$, is 7 [min], the maximum allowable cruising time, W_v^{max} , is 6 [min], and the allowable difference between the wage and fare in the CDP scenario is set to $\Delta = 10$ [\$]. Recall that the optimized time-varying fare and wage (which are the output of the open-loop optimization) are applied to the discrete-event model (i.e., plant in the MPC framework). Agents in the discreteevent model, i.e., riders and drivers, are generated stochastically (both in time and space) based on the rates in Eqs. (6) and (9). The search method of vacant vehicles is the greedy dispatch model where each vacant vehicle (or waiting rider) is matched to the nearest waiting rider (or vacant vehicle), see Section 2. The arrival rates of riders and vehicles depend on the generalized costs that comprise the instantaneous fare and wage and the rider waiting time and vehicles cruising time. These variables, $w_{\rm p}(t)$ and $w_{\rm v}(t)$, are estimated based on the traffic states within the discrete-event model. Whereas the open-loop optimization in the MPC framework uses the developed macroscopic model, i.e., Eqs. (5) and (8). This configuration is a challenge as we can examine the performance of the proposed controller while there are intrinsic differences between the reality and the model.

The optimal pricing strategies are presented in Fig. 7 for each of the four cases. With the unconstrained dynamic pricing (UDP) strategy, the wage temporarily surpasses the fare which does not happen in the other two policies. The constrained dynamic pricing (CDP) strategy also exhibits surge-pricing where both the fare and wage reach their maximum values and become constant before going back down again. In the static pricing strategy, the prices are fixed as defined by the optimization problem.

All four cases exhibit similar pricing patterns. In Cases I, III, and IV, the wage is temporarily higher than the fare under the UDP strategy. In contrast, the UDP in Case II forces the wage to surpass the price more intensely than the other cases to make up for the shortage of the drivers. The CDP strategy also exhibits similar patterns in all four cases, except that in Case III, the maximum wage and prices are attained later than the other cases to cover the driver shortage. In the static pricing strategies, the wage is always lower than the fare to ensure a non-negative overall profit. The profit with the UDP strategy is the highest of the three. For Case I, the profit is 12% higher than the UDP strategy and 30% higher than static pricing, thus demonstrating the efficiency of unconditional control of fare and wage in ride-sourcing markets to increase the profit.

We further observe in Fig. 7 that the CDP strategy has a longer surge-pricing period compared to the other two strategies; the fare and wage are at their maximum value from minute 50 up to minute 130 for Case I. In the UDP strategy, however, the fare and wage have longer ascending and descending periods which allows for better market segmentation and improved revenue management. The fare and wage patterns in Fig. 7 are similar among the pricing

ARTICLE IN PRESS

M. Nourinejad and M. Ramezani/Transportation Research Part B xxx (xxxx) xxx



Fig. 7. The outputs of the proposed MPC controller. The optimal fare and wage for the three pricing strategies. Note the surpass of the wage over the fare with the UDP strategy. Each row represents one case and each columns represents one pricing strategy.

M. Nourinejad and M. Ramezani/Transportation Research Part B xxx (xxxx) xxx





Fig. 8. (a): The cumulative arrival of riders to the ride-sourcing market for different pricing strategies. (b): The fleet size (vacant vehicles plus occupied vehicles) for different pricing strategies.

strategies as the fixed fare and wage in Fig. 7(c) is quite close to the average of time-varying fares and wages with UDP and CDP strategies in Fig. 7(a) and (b). The same applies to the other three cases as well. Hereafter, we further explore the dynamics of the ride-sourcing market for Case I. Fig. 8(a) shows the cumulative number of riders who enter the ridesourcing market. This depends on the pricing strategy as the fare directly affects the proportion of demand that choose the ride-sourcing mode. In addition, the rider waiting time that is a function of the number of waiting riders and the number of vacant vehicles is also influential. Evidently, the UDP strategy attracts more riders to the ride-sourcing market that eventually leads to higher total profit. Fig. 8(b) shows the fleet size (consisting of occupied and vacant vehicles) for different pricing strategies. The results reveal the time-varying fleet size which cannot be captured with equilibriumbased modeling approaches. The UDP strategy is successful to keep the fleet size consistent while CDP and static pricing strategies clearly over supply the market at the beginning and at the end of the control period. More importantly, is the performance of control strategies at the demand peak period which is the supply shortage peak (see minute 90 in Fig. 6). The UDP strategy is effective to keep the vehicle fleet balance throughout the control period specifically around time minute 90 whereas the other two strategies fail and a shortage of vehicles is evident. Fig. 9 displays the time evolution of the states of the system for each pricing strategy. With the UDP strategy, the number of occupied vehicles is noticeably higher than the other two strategies, thus causing a more prolific market for the drivers and eventually the ride-sourcing service provider. In addition, the UDP strategy exhibits less fluctuations in the number of vacant vehicles compared to the other two strategies. The fluctuations in the number of vacant vehicles is the highest with the static pricing. The static pricing also leads to the highest peak in the number of waiting riders that exacerbates the rider waiting times (see Fig. 9(c)). This is because this strategy does not replicate surge-pricing. Hence, the degree of control is minimal.

Fig. 10 displays the waiting time of riders, cruising time of vehicles, and the meeting rate. As mentioned, the rider waiting time is stable during the control period with the UDP strategy and is always below 5 min. The other two strategies, however, violate the desirable waiting time constraints; the static pricing strategy has the highest violation of an additional 4 min. The reason for the violation lies in the difference between the plant and the controller as shown in Fig. 5. The controller only foresees the impact of the pricing strategies in the future of system using the prediction model. In contrast, the plant is a more detailed agent-based and event-driven model of the ride-sourcing market where the predictions are not always with the utmost accuracy. Nevertheless, the results display reasonable synergy between the controller and the plant and demonstrate the applicability of the proposed model and pricing control approach.

The vacant vehicle cruising time is below 7 min in all strategies as required by the controller, see Fig. 10. The UDP strategy results in a noticeable peak (around minute 100) in the vehicle cruising time which forms because of the substantial difference between the fare and wage as permitted in this strategy for the purpose of profit maximization. See Fig. 7(a) where the controller sets the wage higher than the fare around minute 100. This allows the controller to attract more vehicles to the market which initially increases the number of vacant vehicles and subsequently increases the meeting rate (Fig. 10(a)) and the number of occupied vehicles (Fig. 9(a)). It is depicted in Fig. 10(a) that the highest meeting rate is consistently achieved by the UDP strategy with up to 35 boardings per minute. The other two strategies show somewhat stable cruising times and fewer meetings. The results clearly indicate that the proposed model is capable of capturing the time-varying dynamics of the ride-sourcing market where the fleet size changes drastically over time. The MPC controller with the UDP strategy (by setting the wage higher than the fare) leverages the potential in the supply of

ARTICLE IN PRESS

M. Nourinejad and M. Ramezani/Transportation Research Part B xxx (xxxx) xxx



Fig. 9. Evolution of system state, the number of waiting riders, the number of vacant vehicles, and the number of occupied vehicles with the three pricing strategies.

the market and attracts more vehicles to the market to serve the future demand to increase the long-run profit. Fig. 11 presents the generalized cost of riders and vehicles and the profit of the service provider for the three pricing strategies. The generalized costs (to a high extent) resemble the pricing profile of each strategy as depicted in Fig. 7. The difference between the generalized costs and the fare and wage is related to the performance of the market associated with the rider waiting time and vehicle cruising time. We note that the results in Figs. 9-11 are the outputs of the MPC plant (i.e., the agent-based discrete-event model) while the controller (i.e., optimization model of MPC) are based on model (3)-(12a). The profit profiles show that the controller with UDP strategy generates negative profit during the peak of the demand (around minute 100). The negative profit is a result of the wage being slightly higher than the fare to attract more vehicles to enter the market as shown in Fig. 7. Although the UDP strategy displays temporary losses of profit, it ultimately leads to an overall higher profit than the other two strategies. Table 1 presents the performance, i.e. the profit of the ride-sourcing firm, in the four cases under each pricing strategy. The UDP profits are larger than the other strategies in all cases. In Case II, the UDP and CDP attain a higher profit but the static pricing strategy yields a lower profit compared to the other cases because a constant wage and price cannot accommodate and exploit the opportunities of large fluctuations of demand and supply. Case III and Case IV perform slightly better than Case I because the drop in supply and jump in demand does not happen simultaneously, thus allowing the operator to implement more profitable price and wage profile.

M. Nourinejad and M. Ramezani/Transportation Research Part B xxx (xxxx) xxx



Fig. 10. Rider waiting time, vacant vehicle cruising time, and meeting rate with the three pricing strategies.

Table 1

Ride-souring operator profit of the four cases measured in dollars. The values in parenthesis show the improvement with respect to the Static pricing strategy.

	Unconstrained dynamic pricing (UDP)	Constrained dynamic pricing (CDP)	Static pricing
Case I	28,922 (30%)	25,752 (16%)	22,264
Case II	30,619 (39%)	28,415 (29%)	21,981
Case III	29,194 (28%)	27,653 (22%)	22,691
Case IV	29,573 (31%)	26,045 (15%)	22,565

ARTICLE IN PRESS

M. Nourinejad and M. Ramezani/Transportation Research Part B xxx (xxxx) xxx



Fig. 11. Generalized cost of riders and drivers, and the service provider's profit under three pricing strategies.

6. Conclusions

This paper has presented an analytical parsimonious model of non-equilibrium ride-sourcing systems that consider the dynamics of accumulation of waiting riders, vacant ride-sourcing vehicles, and occupied ride-sourcing vehicles. The drivers are modeled as earning-sensitive, self-scheduling, and independent contractors and the riders are considered as price- and quality of service-sensitive such that the supply and demand of the ride-sourcing market are endogenously dependent on the fare, the wage, the rider's waiting time, and the driver's cruising time. We integrate this model into a model predictive control (MPC) approach to optimize the time-varying fare and wage, maximizing the overall profit of the service provider and ensuring a predefined level of service (e.g., keeping the rider waiting time below a threshold). A major contribution of the paper is to use two fundamentally distinct models within the MPC approach. The open-loop optimization is based on the introduced parsimonious macro-model while the plant is a discrete-event micro-simulation model that represents a detailed two-sided ride-sourcing market. The proposed model enables us to investigate dynamic pricing schemes with a time-varying fleet size.

Three pricing strategies are investigated, (i) time-varying and unconstrained, (ii) time-varying and constrained so that the instantaneous profit is positive, and (iii) time-invariant and fixed throughout the control period. We perform numer-

M. Nourinejad and M. Ramezani/Transportation Research Part B xxx (xxxx) xxx

ical experiments to assess the three pricing strategies during a peak period when the demand increases and the supply decreases. We show that the first strategy (dynamic and unconstrained) provides the highest overall profit. Furthermore, this dominant strategy provides a more stable rider waiting time compared to the other strategies. The proposed dynamic modeling and optimal pricing control strategy allow the wage offered to the drivers to surpass the fare collected from the riders, which might seem counter-intuitive in a short-sighted ride-sourcing system design, but this has been demonstrated to be beneficial for the total profit over long-term horizons. The proposed model overcomes the restricting assumptions of the conventional equilibrium-based approach of modeling ride-sourcing markets that is deficient when there are noticeable variations in the state of the system within a short time-frame; the system does not have enough time to balance back into equilibrium.

A crucial future research direction is to investigate more comprehensive supply models of vacant ride-sourcing vehicles that can be validated with field data. The models should address the arrival and exit rates of vacant ride-sourcing vehicles considering the uncertainty in driver perception of future states of the market and the diversity in driver working strategy since drivers can be categorized as duration-based, income-based, or profit-based workers. For further research, the model can be extended to multi-modal multi-firm networks. This is challenging because of intricate interactions between different modes and complex competition dynamics between different ride-sourcing firms. Addressing the effect of temporal propagation of congestion on the market dynamics is an important issue to be tackled in future studies. Another research direction is to model the dynamics of the market in a day-to-day framework, where people adapt and show loyalty towards a specific firm over days. The behavior of users is affected by their previous experiences of paid fare, waiting time, and total travel time that are a function of the pricing control strategy. Furthermore, validation of the dynamic meeting function and the model with real world data would offer more insights about the implications and applicability of the proposed model and control strategy.

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18

M. Nourinejad and M. Ramezani/Transportation Research Part B xxx (xxxx) xxx

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