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Staggered work schedules for congestion mitigation: A morning commute problem☆

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A R T I C L E I N F O

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A B S T R A C T

In urban networks, traffic congestion can be curbed by deconcentrating the temporal distribution of the travel demand. In this paper, we propose an optimal staggered work schedules problem to minimize the network total travel time and prevent the schedule delay in the trips of commuters over morning peaks in a bicentric network. The objective is to optimize the work start times of individual firms with minimum deviations from their initial schedules while taking into account that commuters choose their departure time selfishly to minimize their travel cost. We formulate the optimal work schedule problem in a bicentric network as a multi-objective optimization program that simultaneously minimizes the total travel time and the schedule deviation for the firms while satisfying near-equilibrium temporal conditions. The time-varying congestion dynamics are modeled using macroscopic fundamental diagrams. We solve the optimization problem for a test network and analyze the sensitivity of the Pareto solution to the policy parameters of the model. We assess the accuracy and effectiveness of the proposed method using an individual-level trip-based macroscopic simulation model. The numerical results demonstrate that implementing the proposed optimal staggered work schedules strategy accounting for commuters' departure trip time choice can significantly reduce the traffic congestion in urban networks.

1. Introduction

Traffic congestion counts as a critical issue in large cities, one that imposes a tremendous burden on the society, economy, and environment. The travel demand management policies aim to alleviate congestion by altering the travel behavior of network users. However, implementing policies that require penalizing the travelers during the peak periods faces social resistance and technical challenges that significantly limit the effectiveness of the user-based policies, e.g., congestion pricing. The staggered work schedules is a practical alternative to the user-based policies that indirectly alter the travel behavior of the users by shifting the schedules of their activities at destinations.

The trip scheduling problem was introduced in Vickrey (1969) for the morning commutes through a first-in, first-out single bottleneck with a fixed capacity. Commuters wish to arrive at their destinations punctually; however, it is physically impossible for

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everyone to be on time when the demand exceeds the capacity. By adjusting the start time of their trips, rational commuters minimize the combined cost of congestion delay (the difference between actual and free-flow travel times) and schedule deviation (the difference between actual and wished arrival times) associated with their trips. The cumulative result of the individual trip start time decisions can lead to an equilibrium state (under some conditions) in which no commuter can reduce their combined travel cost by changing the start time of the trip. Vickrey's congestion theory has been further elaborated in the literature by accounting for the heterogeneity in the wished schedule and schedule penalty preferences of the commuters (Henderson, 1974, 1981; Hendrickson and Kocur, 1981; Arnott et al., 1992, 1994; van den Berg et al., 2011; Liu et al., 2015; Wu and Huang, 2015; Amirgholy and Gonzales, 2017; Long and Szeto, 2019). The existence and uniqueness of the equilibrium solution is also proved for a general distribution of the wished schedules in Smith (1984) and Daganzo (1985), and for users with heterogeneous schedule penalty preferences in Lindsey (2004).

The morning commute problem has been also studied in urban networks with an aggregated traffic model, i.e., macroscopic fundamental diagram (MFD). The concept of MFD with an optimum accumulation is initiated by (Godfrey, 1969). Similar ideas are later introduced by (Herman and Prigogine, 1979; Mahmassani et al., 1984; Daganzo, 2007). The empirical existence of MFD is shown by (Geroliminis and Daganzo, 2008). Recently, Loder et al. (2019) provided further empirical evidence on the existence of MFD from many cities around the world. MFD provides a unimodal, low-scatter, and demand-insensitive relationship between network-wide traffic states (e.g. accumulation, speed, production, and trip completion flow) for an urban region. A number of models have been developed to estimate MFD considering limited loop detector data (Saffari et al., 2020), probe vehicle data (Leclercq et al., 2014) or both loop detector and probe vehicle data (Ambühl and Menendez, 2016). MFD has offered the possibility of designing network-level traffic management and control schemes. Some examples are perimeter control in urban networks (Aalipour et al., 2018; Su et al., 2020; Ingole et al., 2020), regional route guidance (Yildirimoglu et al., 2018), and pricing strategies (Zheng et al., 2016; Gu et al., 2018). MFD has also been studied in the context of dynamic traffic assignment models and several methods have been proposed; a numerical algorithm focusing on multiple regions (Yildirimoglu and Geroliminis, 2014; Batista and Leclercq, 2019), a single origin-destination pair with two alternative routes (Laval et al., 2018), a continuum approximation model (Aghamohammadi and Laval, 2020), and an assignment model with multiple modes (Liu and Geroliminis, 2017; Petit et al., 2021).

Combining Vickrey's congestion theory with MFD modeling makes it possible to model the morning commute problem in urban networks, in which the network outflow can be formulated as a function of the instantaneous vehicular accumulation, e.g. see Geroliminis and Levinson (2009). However, dependence of the network outflow at each point in time upon prior states of traffic makes the problem intractable (Arnott, 2013). Various studies make different assumptions to solve tractable versions of the network problem: the users' travel times in the network depend only on the vehicular accumulation at the time of their arrivals to the network (Small and Chu, 2003; Geroliminis and Levinson, 2009); the MFD is a two-step function (Fosgerau and Small, 2013); the probability that individual users exit the network at each point in time is directly proportional to the region speed and inversely proportional to their trip length (Arnott, 2013); the MFD can be represented by Greenshields' relation (Arnott et al., 2016); the travel time of users remains constant off the peak (Amirgholy and Gao, 2017). Alternatively, the exact solution of the network problem can be numerically estimated by a trip-based simulation model (Arnott, 2013; Mariotte et al., 2017; Lamotte and Geroliminis, 2018; Sirmatel et al., 2021). These efforts can be used to design demand management policies for large urban networks (Yildirimoglu and Ramezani, 2019; Kumarage et al., 2021). While there are many studies focusing on equilibrium properties, less attention has been given to how the resulting framework, involving both Vickrey's theory and MFD modelling, can be useful to develop staggered work schedules, aiming to alleviate traffic congestion in the network.

The staggered work schedules is a demand management policy that deconcentrates the distribution of the work start times with the aim of spreading the peak travel demand. The idea of alleviating the rush-hour traffic congestion by flattening the peak demand has been widely discussed in the literature (Henderson, 1981; D'Este, 1985; Zhang et al., 2005; Mun and Yonekawa, 2006; Takayama, 2015; Shabanpour et al., 2017; Li et al., 2017). The underlying theory is based on a central principle of microeconomics that rational agents make efficient decisions when faced the full social consequences (benefits and costs) of their actions (Arnott et al., 2005). In urban networks, work trips make up the majority of the morning travel demand. The congestion delay can then be minimized by incentivizing the firms to make efficient choices of work start times. The incentivizing policies can be implemented in various forms, e.g., time-varying wage rates for workers (Henderson, 1981; Wilson, 1988), time-dependent tax deductions for firms (Arnott et al., 2002; Amirgholy and Gao, 2017), and compensation payments made to employers for each reassigned employee (Yushimito et al., 2014; Yushimito et al., 2015). Even though the changes in work schedules have usually been implemented with ad hoc strategies in practice, they have been shown to be effective in reducing the traffic congestion in large cities such as New York (O'Malley, 1974), Toronto (Greenberg and Wright, 1974), Ottawa (Safavian and Mclean, 1975), Singapore (Wilson, 1988), Honolulu (Giuliano and Golob, 1990), Geneva (de Palma et al., 1998), and Brisbane (Cleary et al., 2010). On an aggregated level, the cumulative result of the efficient work start time choices of individual firms should lead to an optimal distribution of the travel demand over the peak, and alleviate traffic congestion in the whole network. Nonetheless, the existing studies mostly focus on small-scale networks with a limited number of employers to consider, and overlook the complex interaction between a large number of employers located in various zones and districts of the network. This paper fills this gap by developing a systematic approach to tackle staggered work schedules problem in bicentric large-scale networks considering the employers (firms) constraints.

In this paper, we propose an optimal staggered work schedule strategy (for bicentric cities) that involves multiple criteria. A good design of the optimal work schedules should (i) minimize the total travel time in the network (ii) under near-equilibrium conditions (iii) with minimal changes in the initial work schedules of employers. Essentially, the goal is to mitigate the traffic congestion through the changes in the work schedules of firms (i.e. employers of commuters). However, an optimal design of the work schedule must also consider the new collective traffic conditions that occur when commuters change their departure times in response to changing work schedules. In other words, the optimal schedule design must be consistent with (near) equilibrium conditions, which approximately captures travellers' response to changing schedules. Additionally, the optimal design should not significantly deviate from the initial work schedules and yield as minimal changes as possible, because implementing substantial changes in work schedules may face resistance from both employers and employees. The two objectives, minimizing total travel time in network and minimizing the deviation from the initial firms work schedules, may be conflicting in nature, which calls for a multi-objective optimisation framework where each objective can be represented as a weighted term. We evaluate the Pareto solution of the optimization problem by performing a sensitivity analysis on the weight terms of the model. Furthermore, the impacts of the work schedules and commuters' departure time choice on traffic conditions (under user equilibrium conditions) are captured by an accumulation-based bathtub (MFD) model, which allows an elegant optimization structure for this large scale problem.

The proposed framework further accounts for multiple employers with various work schedules, and includes a lower-level (employer-level) optimisation problem that decomposes the optimal (regional) work schedule to work start times of different firms in each region. The objective of the employer-level problem is to optimize the work schedules of the individual firms with minimum deviations from the desired schedules. To provide a numerical experiment, we solve the optimization problem for a bicentric network and present the set of optimal work schedules for the firms. Noteworthy, we develop a detailed trip-based MFD simulation model to assess the accuracy and effectiveness of the proposed approach. The trip-based model takes into account the characteristics of individual commuters such as trip length, wished arrival time, and lateness and earliness willingness. The results indicate that the congestion in the network can be substantially curbed with small changes in the work schedules while satisfying equilibrium conditions.

In this paper, we consider and envision a city with two metropolitan centres offering a concentration of jobs and a wide range of goods and services. We acknowledge that the era of the traditional city, with one central business core surrounded by residential rings of low density, may no longer be sustainable; cities are evolving toward a polycentric, multi-nodal model. For instance, Greater Sydney Commission has developed a 30-min city concept with three major centers as the centerpiece of its 40-year plan. The aim is all Sydneysiders to be able to reach one of the three metropolitan centers in less than half an hour. The move towards such polycentric structures will also require the design of work hours, which this paper aims to achieve through a systematic approach. Note that, the assumed bicentric network configuration is not meant to be representative of all cities; the proposed framework can be adjusted and extended to incorporate other polycentric city scenarios.

The remainder of the paper is organized as follows: Section 2 introduces an overview of the proposed framework. Section 3 elaborates the main principles for the modelling of the bicentric urban network. In Section 4, we present the mathematical formulation of the optimization problems at the network-level and employer-level. Section 5 evaluates the results of the numerical experiments and the trip-based simulation model. Lastly, conclusions of the paper are summarized in Section 6.

2. Overview of the methodology

In urban networks, the rush-hour traffic congestion can be alleviated by spreading the temporal distribution of the demand over the peak period. The objective of the proposed optimal work schedule strategy is to (i) minimize total time spent (or congestion) in the network (ii) with minimal changes (or deviation) from the original work schedules (iii) while taking into consideration travellers'

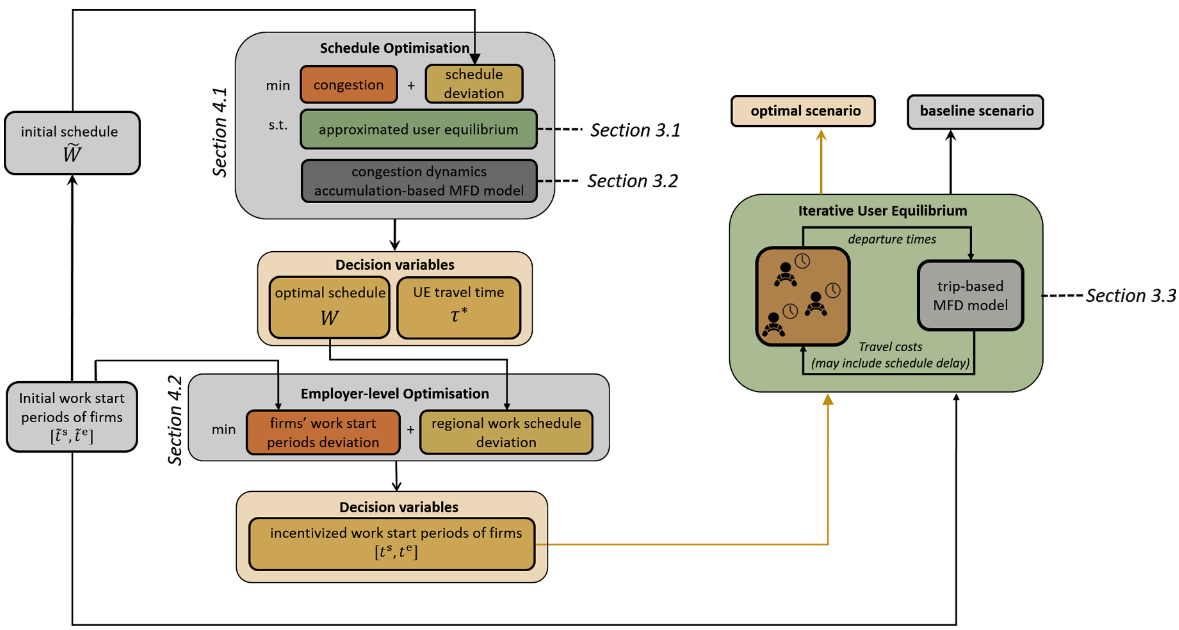

Fig. 1. The structure of the proposed methodology.

departure time choices (i.e., user equilibrium conditions). This schedule optimisation strategy can be formulated as an (abstract) optimization problem, see 'Schedule Optimisation' in Fig. 1. While the objective is to minimize congestion and schedule deviation, the resulting solution should satisfy (approximated) user equilibrium conditions and congestion propagation dynamics in the network. In this study, we model the congestion dynamics of the bicentric network on a macroscopic level as a multi-reservoir MFD-based queuing system with time-dependent demand and state-dependent capacity, i.e., accumulation-based MFD. Note that this approach is quite different from the single-bottleneck models, which assume capacity is constant, or other MFD-based morning commute models, which assume that trip scheduling problem arises only after the system exceeds the critical accumulation. In this queuing system, rational users seek to minimize their own travel costs in the network by adjusting the start times of their trips subject to adherence to the work start time of their firm (this can be an interval, for instance 8:00 am - 8:30 am). The cumulative result of the individual trip scheduling decisions leads to temporal equilibrium or user equilibrium (UE) conditions at the network level in which no one can reduce their travel cost by shifting the start time of their trip. Nonetheless, given the state-dependent capacity feature of the accumulation-based MFD model, it remains a challenge to find an analytical solution to UE conditions in multi-reservoir networks. Note that this is the case despite all the commonly adopted assumptions on first-in-first-out behavior and constant trip length (i.e., all users in the same reservoir travel the same distance).

Therefore, there is need for further simplification in relation to the target UE solution so that an analytical relation can be derived within the optimization problem. Hence, we define 'approximated user equilibrium' conditions, as shown in Fig. 1. Considering the fact that this is a schedule design problem, we opt for a scenario where everyone arrives at work on time and no one suffers a schedule cost. The opposite would imply that, even with the optimal schedule scenario, the design capacity is not enough for commuters to arrive on time and some have to arrive at their destination earlier or later than they wish. This clearly does not indicate ideal conditions that one should be seeking when designing optimal work schedules. However, this approach (imposing a zero-schedule delay approximation) does not guarantee that the final solution is the best solution in terms of the total cost (i.e., sum of queuing cost and schedule cost); there may be other scenarios where the total cost is less albeit producing non-ideal conditions. Note that the aforementioned zero-schedule cost approximation is adopted only at the aggregated design stage (i.e., schedule optimisation in Fig. 1)) to generate the optimal work schedules; the resulting solution will later be put to the test considering more realistic UE settings (i.e., 'iterative user equilibrium' as shown in Fig. 1) where individual users can make individual trip scheduling decisions and a schedule cost may arise as a result of collective decisions in the network.

The decision variables in the schedule optimisation problem are the work schedules and UE travel times, see Fig. 1. UE travel times indicate the total costs that commuters experience considering zero-schedule cost approximation and can be considered a by-product of the optimisation framework, whereas the optimal work schedule is the actual outcome of the optimisation framework that needs to be further tested considering employer-level schedules and more realistic UE settings. The proposed staggered work schedule strategy comprises of two levels; in the top level aggregated region-level schedules are considered and optimised, while at the employer level, the aggregated optimal work schedule is broken down into to the work start period of individual firms (employers). In other words, we consider a bi-scale formulation where the congestion is curbed with a travel demand management method that alters the work start period of individual firms.

Ultimately, the optimal work start periods of firms are incorporated into iterative UE module, where commuters make departure time (or trip scheduling) decisions with respect to travel costs (including schedule costs) they experience in the traffic system, see Fig. 1. The decision update mechanism here is modelled as a day-to-day assignment model, and the system dynamics of the bicentric network are modelled using a trip-based MFD model. Intrinsically, this model relies on an aggregated traffic performance function (i.e., speed-MFD), but unlike the accumulation-based MFD, it allows the modelling of individual travellers with distinct characteristics such as lateness and earliness willingness and individual trip decisions. Hence, it is a suitable model to retrieve individual travel costs (including travel time and schedule costs) and to establish the 'actual' UE conditions that result from the work schedule that has been designed by the proposed schedule optimisation program. Note that the same UE module is used to build the baseline scenario too,

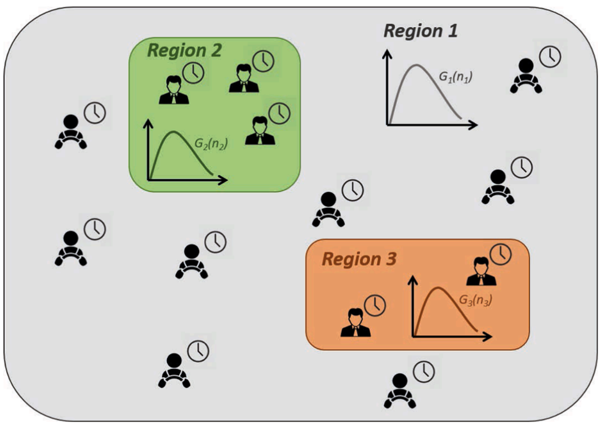

Fig. 2. Schematic of the bicentric urban network. Trips originate in residential Region 1 with destination in downtown Regions 2 or 3 where the firms are located.

where the work schedule is the baseline schedule representing the initial conditions.

3. Main principles for the modelling of the bicentric urban network

The bicentric structure of the network in this queuing system allows us to consider that the trips originated in the residential Region 1 have destinations in either of the downtown Regions 2 or 3, surrounded by Region 1, as illustrated in Fig. 2. Note that our scenario does not include other types of demand (e.g., from Region 2 to Region 1) which should be insignificant in the morning commute of this bicentric network. In this model, the travel time that users bear in their commutes to Region $i \in \{2, 3\}$ also depends on the trip schedule of the commuters to Regions $j \neq i, j \in \{2,3\}$, since their trips have an overlap in Region 1. We capture the dynamics of the congestion in the bicentric network by an accumulation-based bathtub model in which the outflow of Region 1 is the inflow to Regions 2 and 3. The interrelationships between the region outflow and accumulation in each region is represented by the MFD. We simultaneously optimize the work schedule of the commuters to Regions 2 and 3 satisfying both temporal equilibrium conditions and system dynamics.

In the proposed modeling framework, the morning trips have an origin in the residential suburban area, Region 1, and a destination in one of the city centers, Regions 2 or 3, please see Fig. 2. Thus, it is reasonable to assume that commuters directly travel to their final destinations in Region 2 or 3, without passing through a third region. Nevertheless, to account for small number of travellers who might prefer to cross a third region in reality or pass the region boundaries more than one time, traffic assignment models (route choice) aiming for equilibrium conditions in the context of MFD modelling could be incorporated as a future research (Yildirimoglu and Geroliminis, 2014; Guo and Ban, 2020).

The proposed framework builds on several key principles: (i) approximated user equilibrium definition, (ii) congestion propagation dynamics or accumulation-based MFD modelling, and (iii) iterative user equilibrium approach, which will be introduced in the following subsections. Please refer to Fig. 1 for their use in the proposed framework.

3.1. Approximated user equilibrium

Commuters starting their trips in Region 1 wish to arrive at their destinations in Regions 2 and 3 punctually. However, in certain scenarios, the rapid decline in the network speed with the rise of the accumulation in Regions 1, 2, and 3 over the morning peak can make it physically impossible for everyone to be on time. Thus, the commuters may experience longer travel times in the network and arrive at their destinations earlier or later than they wish. Total cost in the system, including travel times and schedule cost, can be significantly curbed by flattening the distribution of the wished arrival times or work schedules. The schedule optimization scheme that we propose in this study aims for a scenario where all commuters arrive on time and no schedule cost is observed. In this case, all the users arrive at their destinations in regions 2 and 3 on time experiencing the minimum delay in their trips. Bearing no schedule cost, users seek to minimize their own travel times by adjusting their trip schedules. Thus, under the equilibrium condition, the travel time remains equal for each and every individual user who has a destination in the same region, i.e., region 2 or 3. In other words, our target scenario is the one where all travellers (with the same destination) have the same travel time with zero schedule cost; this way, no traveller would have an incentive to change their departure time and induce schedule cost. Note that if travel time is the same for every commuter, changing departure time would simply cause a schedule cost that did not exist in the original departure time choice.

In a single bottleneck or a single region context, the change in travel times during the peak hours has to be accompanied with changing schedule costs in order to satisfy equilibrium conditions and guarantee that no one can improve their cost. However, this is not needed in the bicentric network that we consider; the increase in travel time in one region may be compensated by the decrease in travel time in the other region such that travel times are approximately equal for all travellers. Note that the single bottleneck and the single region bathtub models assume that the travel time is constant under uncongested conditions, and the schedule cost appears once the system exceeds the capacity flow. Nevertheless, Yokohama MFD presented in Geroliminis and Daganzo (2008) shows that average

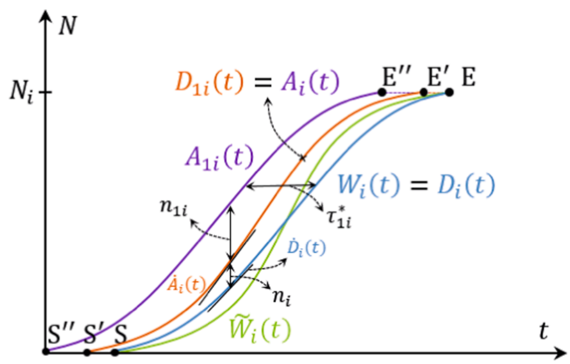

Fig. 3. The queueing diagram of the bicentric urban network with an optimal wished curve. $W_i(t)$ is the optimal cumulative distribution of the staggered-wished arrival times at Region $i \in \{2,3\}$ that is designed to be equal to the cumulative distribution of their actual departure times from Region $i, D_i(t)$. This is a deliberate design approach such that the schedule cost is zero.

speed in the network changes from approximately 30 km/h at zero accumulation to 15 km/h at the critical accumulation (brink of congested regime). In this study, we relax the 'constant travel time' assumption for the uncongested regime and account for variations in travel time in the whole range of accumulations.

Now, let us elaborate the approximated user equilibrium conditions that we target to have in the proposed optimal work schedule scenario. Note that this discussion does not account for an arbitrary work schedule and corresponding equilibrium conditions; it refers specifically to the proposed optimal work schedule distribution and the corresponding UE conditions where schedule cost does not exist. Having zero-schedule cost in the approximated UE scenario will allow us to develop an elegant formulation of the congestion dynamics and UE conditions. It is envisaged that the optimal work schedule distribution will yield a scenario in which all the commuters experience equal travel times to Region $i \in \{2,3\}$ during the peak period, i.e., equilibrium travel time, τ^*_{1i}:

$$\tau_{1i}(N) = \tau^*_{1i} \quad \text{for} \quad N \in [1, N_i] \tag{1}$$

where N_i denotes the total number of commuters with destination in Region i and $\tau_{1i}(N)$ is the travel time of commuter N from Region 1 to i, which can be expressed as the summation of the travel times of commuter N through Regions 1 and i, $\tau_1(N)$ and $\tau_i(N)$ respectively: $\tau_{1i}(N) = \tau_1(N) + \tau_i(N)$.

The queuing diagram of Fig. 3 illustrates the UE conditions in the bicentric network for an optimal distribution of the wished arrival times. In this queuing diagram, $\widetilde{W}_i(t)$ is the cumulative distribution of the initial work schedule of employers in Region i, i.e., the cumulative distribution of the wished arrival times of the commuters at their destinations in Region i in the initial scenario. In contrast, $W_i(t)$ is the optimal cumulative distribution of the wished arrival times of the commuters at their destinations in Region i resulting from the schedule optimization problem, please see Fig. 1. Note that $\widetilde{W}_i(t)$ is depicted in the diagram for comparison to $W_i(t)$; the remaining curves/variables belong only to the optimal work schedule scenario. For users to bear no schedule cost in their commutes in the optimal work schedule scenario, the cumulative counts of the wished arrivals at destinations in Region i by time t, $W_i(t)$, should be exactly equal to the cumulative counts of the actual departures from Region i (or the actual arrivals at destinations in Region i) by time t, i.e., $D_i(t) : W_i(t) = D_i(t)$. Considering the bicentric network structure presented in Fig. 2, the cumulative counts of the arrivals at Region i by time t,$A_i(t)$, is equal to the cumulative counts of the departures from Region 1 by time t that have a destination in Region i, $D_{1i}(t) : A_i(t) = D_{1i}(t)$. $A_{1i}(t)$ is the cumulative counts of the arrivals by time t in Region 1 with a destination in Region i. With this definition of the arrival and departure curves, the travel time that users bear in their commutes from Region 1 to their destinations in Region i can be graphically presented as the horizontal distance between the cumulative arrival curve in Region 1, $A_{1i}(t)$, and the cumulative departure curve from Region i, $D_i(t)$. For the equilibrium condition to hold (Eq. 1), the horizontal distance between these curves should remain exactly equal for all the users over time: $\tau^*_{1i} = D_i^{-1}(N) - A_{1i}^{-1}(N)$ for $N \in [1, N_i]$. In other words, $A_{1i}(t)$ should be a translate of $W_i(t)$ or $D_i(t)$ to the left of t-axis by τ^*_{1i}. Nevertheless, it is obvious that the arrival and departure curves introduced here are also a function of congestion dynamics, which will be introduced in the next subsection. The arrival (and departure) curves resulting from congestion dynamics and resulting from user equilibrium principles may not be consistent with each other. That means, $A_{1i}(t)$ resulting from the approximated user equilibrium defined in Eq. eq:eql may not satisfy the congestion dynamics that will be presented in the next subsection. This will be further discussed in Section 4, particularly with respect to the formulation of the optimisation problem.

3.2. Congestion dynamics - Accumulation-based MFD model

We consider an accumulation-based bathtub model to capture the traffic dynamics in the bicentric network. In this model, the relationship between the network outflow and the vehicular accumulation is determined by the MFD. Let us assume that the urban network is partitioned into three regions each with well-defined MFDs, as shown in Fig. 2. Let $n_{1i}(t)$ [veh] denote the vehicle accumulation in Region 1 with destination in Region $i \in \{2,3\}$ at time t. Also let $n_i(t)$ [veh] be the total accumulation in Region $i \in \{2,3\}$ at time t. Evidently, $n_1(t) = n_{12}(t) + n_{13}(t)$.

Considering the approximated UE conditions described in Eq. 1, the traffic flow conservation equations of the bicentric urban network are as follows:

$$\frac{\mathrm{d}n_{1i}(t)}{\mathrm{d}t} = \dot{A}_{1i}(t) - \dot{D}_{1i}(t) \quad i \in \{2,3\} \tag{2}$$

$$\frac{\mathrm{d}n_i(t)}{\mathrm{d}t} = \dot{A}_i(t) - \dot{D}_i(t) = \dot{D}_{1i}(t) - \dot{W}_i(t) \quad i \in \{2,3\} \tag{3}$$

where $\dot{A}_{1i}(t)$ is the slope of the arrival curve to Region 1 with final destination in Region i at time t, $\dot{D}_{1i}(t)$ is the slope of the departure curve from Region 1 to Region i at time t, $\dot{A}_i(t)$ is the slope of the arrival curve to Region i, $\dot{D}_i(t)$ is the slope of the departure curve from Region i, and $\dot{W}_i(t)$ is the slope of the departure curve from Region i (and the network) at time t. The queuing diagram presented in Fig. 3 is consistent with the accumulation-based model defined above. That means, $\dot{A}_{1i}(t)$ is the input travel demand to Region 1 with destination in Region i at time t, and $\dot{D}_{1i}(t)$ is the transfer flow from Region 1 to Region i at time t (hence the negative sign in Eq. 2). Similarly, $\dot{A}_i(t)$ is the input travel demand to Region i at time t, and $\dot{D}_i(t)$ is the exit flow from Region i at time t (hence the negative sign in Eq. 3). Considering the structure of the bicentric urban network, $\dot{A}_i(t) = \dot{D}_{1i}(t)$, and due to zero schedule delay assumption outlined in Eq. 1, $\dot{D}_i(t) = \dot{W}_i(t)$. Therefore, Eq. 3 can be rewritten as $\dot{D}_{1i}(t) - \dot{W}_i(t)$.

Accordingly, the vertical distance between the arrival and departure curves of Region $i \in \{2,3\}$ represents the accumulations of Region i at each point in time, $n_i(t) = A_i(t) - D_i(t)$. Similarly, the accumulation of Region 1 is the difference between the cumulative arrivals and departures of Regions 1, $n_1(t) = A_1(t) - D_1(t)$ where $A_1(t) = \sum_{i\in\{2,3\}} A_{1i}(t)$ and $D_1(t) = \sum_{i\in\{2,3\}} D_{1i}(t)$, in the bicentric network.

Given the MFD of Regions 1 and $i \in \{2,3\}$, $G_1(\cdot)$ and $G_i(\cdot)$, we estimate the region outflow at time t as a function of the accumulation of the region at time t, i.e., $\dot{W}_i(t) = \dot{D}_i(t) = G_i(n_i(t))$. For Region 1, we have $\dot{D}_1(t) = G_1(n_1(t))$, while $\dot{D}_1(t) = \sum_{i\in\{2,3\}} \dot{D}_{1i}(t)$ and $n_1(t) = \sum_{i\in\{2,3\}} n_{1i}(t)$. A further assumption is needed to determine the accumulation components in Region 1. Consistent with the accumulation-based MFD models in the literature, we assume the ratio of outflow from Region 1 to Region i to the total outflow from Region 1 is equal to the instantaneous ratio of accumulations of vehicles in Region 1 with respect to their destinations, i.e. $n_{12}(t)/n_{13}(t) = \dot{D}_{12}(t)/\dot{D}_{13}(t)$.

The above relations describe the congestion propagation dynamics that are needed to estimate the arrival and departure curves in Fig. 3. Let us assume $W_i(t)$ is known and differentiable. As our target optimal scenario does not involve schedule cost, $W_i(t) = D_i(t)$. And, the MFD dynamics requires that $G_i(n_i(t)) = \dot{D}_i(t)$ or $G_i(n_i(t)) = \dot{W}_i(t)$. This simply means the number of vehicles completing their trips at time t is a function of MFD, G_i, and accumulation, $n_i(t)$. Graphically speaking, at each time t, one can find the slope of $D_i(t)$ or $W_i(t)$, and using the inverse of MFD function, derive the corresponding $n_i(t)$, and finally vertically add $n_i(t)$ to $W_i(t)$ to estimate $A_i(t)$. A similar procedure can be applied to estimate $A_{1i}(t)$. MFD of Region 1 requires that $G_1(n_1(t)) = \dot{D}_{12}(t) + \dot{D}_{13}(t)$. Note that the two derivative terms on the right hand side can be numerically computed from the cumulative curves, i.e., $A_2(t) = D_{12}(t)$ and $A_3(t) = D_{13}(t)$. G_1 is known as well. Hence $n_1(t)$ can be estimated. Also by definition $n_1(t) = n_{12}(t) + n_{13}(t)$, and as described above $n_{12}(t)/n_{13}(t) = \dot{D}_{12}(t)/\dot{D}_{13}(t)$. Given these relations, $n_{12}(t)$ and $n_{13}(t)$ can be estimated. And, $A_{1i}(t)$ can be constructed, i.e., $A_{1i}(t) = A_i(t) + n_{1i}(t)$. Essentially, given $W_i(t)$, $A_i(t)$ and $A_{1i}(t)$ can be calculated considering the approximated user equilibrium definition (i.e., $W_i(t) = D_i(t)$) and the outlined congestion dynamics. The above relations are the methodological principles that are mathematically expressed in the optimization problem in Section 4. Note that the queuing diagram presented in Fig. 3 presents an ideal scenario where the arrival curve $A_{1i}(t)$ fully satisfies both approximated user equilibrium principles and congestion dynamics. Nevertheless, such ideal conditions may not be possible to achieve in a real implementation, which will be discussed in Section 4 considering the formulation of the schedule optimisation problem.

Having the components of the accumulation-based model formulated, the staggered work schedule method minimizes the total travel time of all commuters through the regions of the bicentric network by optimizing the wished arrival times $W_2(t)$ and $W_3(t)$ of the commuters at their destinations while considering the behavioral reaction of drivers captured by changes in arrivals to Region 1, $A_{1i}(t)$. Also note that the UE conditions presented in the previous subsection and the congestion dynamics presented above may not result in the same arrival and departure curves, particularly the same A_{1i}. This mismatch will be further discussed in Section 4, and will be added as one of the terms to minimize in the objective function.

Due to its low complexity and analytical tractability, the accumulation-based MFD model presented above is a promising tool to develop network-level traffic management schemes. Nevertheless, this modeling approach does not allow tracking of individual vehicles in the network, and the framework that we build around this model has certain limitations, e.g., constant average trip length ($\bar{l}$), first-in first-out, etc. In order to address some of the limitations, in this work, we will test the optimisation results in a more detailed simulation model which allows tracking of individual travellers. Daganzo and Lehe (2015, 2018, 2017) propose a different model (called the trip-based MFD model) where vehicles might have different trip lengths and move toward their destination with respect to the average speed in the region defined by 'speed-MFD', $V(n)$. The speed-MFD can be expressed as $V(n) = P(n)/n$, where $P(n)$ is the 'production-MFD' that is $P(n) = G(n) \cdot \bar{l}$ in steady-state conditions. We develop the trip-based MFD model of the bicentric network and apply the optimized work schedules to evaluate the network total travel time.

3.3. An iterative algorithm to determine user equilibrium conditions

To address UE conditions that would result from an arbitrary work schedule scenario, we turn over to the trip-based MFD model (more details on this model are provided in the next subsection) and an iterative assignment approach, see the iterative UE module in Fig. 1. Essentially, this approach enables the observation of individual travel costs, including travel times and schedule costs, which are necessary for individual trip scheduling decisions to be made. In this framework, we adopt a day-to-day assignment model where commuters update their departure time decisions considering their perceived travel cost, which is a combination of historical travel cost and current cost. Although it does not guarantee an exact equilibrium solution (or convergence for that matter), this iterative approach approximates the equilibrium conditions in complex system settings such as the bicentric network that we consider in this study. Note that while a numerical approximation of the analytical equilibrium solution exists for single region networks (Amirgholy and Gao, 2017), such solutions are not available for the multi-regional networks, e.g., the bicentric network under study. For more details on the implemented day-to-day assignment model, see (Yildirimoglu and Ramezani, 2019). The iterative UE module is used as the testbed (i.e., the *plant* representing reality) in the proposed framework for two purposes; (i) to identify the baseline scenario considering the initial work schedules, and (ii) to assess and validate the proposed optimal work schedule scenario without the assumptions on approximated UE conditions (i.e., zero schedule delay).

The trip-based MFD model is a disaggregated and network-wide model that takes into account individual commuter characteristics such as trip length, arrival time, and lateness and earliness characteristic. We consider the event-based implementation of the trip-based MFD model for the bicentric network. The events are arrival to Region 1 (departure from the residential place), transfer to Region 2 or 3 depending on the final destination, and departure the network from the destination region (arrival to their working place). The model gets updated when an event occurs. These events are associated with changes in the accumulation of regions (i.e., n_{12}, n_{13}, n_2, n_3) that lead to changes in the speeds of the regions according to the speed-MFD, $V_1(n_1), V_2(n_2), V_3(n_3)$ respectively. Based on the region speed, the traveled distance of all commuters is updated. The vehicles exit the region or the network once their traveled distance becomes equal to their assigned trip lengths. Intuitively, each traveller has two pre-assigned trip lengths, one in Region 1 and one in their destination Region *i*. This iterative and unevenly-clocked procedure continues till all the commuters reach their destinations and their travel times in Region 1 and the destination region is calculated. The pseudo algorithm of the trip-based MFD model for the bicentric network is provided in Appendix A. This modelling approach allows us to estimate the schedule cost of each commuter as the difference between their wished departure time and their actual departure time can be estimated. Note that in schedule optimisation framework we employ the approximated UE conditions to ease the optimization program. Use of trip-based model relaxes this assumption as each traveler has their schedule cost that plays a role in establishment of user equilibrium condition.

4. Formulation of the optimal work schedule problem

The proposed staggered work schedule strategy comprises of two levels; Section 4.1 introduces the top level problem of aggregated schedule optimisation, while Section 4.2 introduces the employer level problem, where the aggregated optimal work schedule is broken down into to the work start period of individual firms (employers). Please see Fig. 1 for the sequencing of the two levels as well as the key principles required in the proposed framework.

4.1. Multi-objective schedule optimisation

Ideally, the objective of the proposed schedule optimization scheme is to minimize the *congestion* cost and the *schedule deviation* (i.e., change from the original work schedules) subject to *equilibrium* conditions and congestion propagation dynamics, as presented in'schedule optimisation' in Fig. 1. Nonetheless, to exactly satisfy *equilibrium* conditions with zero-schedule delay approximation, the horizontal distance between the arrival curve (to Region 1) and departure curve (from destination Region *i*) should remain exactly the same for all users over time: $\tau^*_{1i} = D_i^{-1}(N) - A_{1i}^{-1}(N)$ for $N \in [1, N_i]$, as described in Section 3.1. This is a very strict constraint for the proposed optimization framework to handle. Therefore, we opt to transfer this constraint to the objective function as a penalty term that accounts for the constraint violation. Mathematically speaking, we define a new arrival curve, $\widetilde{A}_{1i}$, that represents the arrival pattern under perfect equilibrium conditions; $\widetilde{A}_{1i}$ should simply be translate of D_i with a lag of τ_{1i}. The added penalty term aims to minimize the difference between the equilibrium arrivals, $\widetilde{A}_{1i}$, and the system arrivals, A_{1i}, which complies with congestion propagation dynamics. Our approach does not guarantee that the two curves will be identical, but ensures they will be as close as possible, i.e. near-equilibrium conditions. Additionally, this penalty term has to be associated with a higher weight than others, given that it refers to constraint violations in the original problem.

In light of the above discussion, we formulate the optimal work schedule problem as a multi-objective optimisation problem considering multiple aspects of the problem in hand. The objective function presented in Eq. 4a consists of three terms that are surrogates for *equilibrium* approximation, *congestion* cost, and *schedule deviation*. Each term is associated with a weight parameter, i.e., α, β, γ, which will be further discussed in the next section. We have combined these terms into one single-objective scalar function using the scalarization method. Alternatively, this problem can be formulated as a bi-level programming or a game theory problem, where the upper level might focus on the allocation of work start times, and the lower level accounts for the response of travellers to the new schedule (satisfying the equilibrium condition). Nonetheless, the scalarization method has been widely used to tackle multi-objective problems, and the solution to this single-objective function is an efficient solution for the original multi-objective problem, i.e. Pareto-optimal solution (Caramia and Dell'Olmo, 2020). Essentially, the first term *equilibrium approximation* indicates a measure of similarity between the arrival curve that arises from system dynamics, i.e., A_{1i}, and the one that complies with equilibrium conditions, i.e., $\widetilde{A}_{1i}$, which is translate of W_i as in Eq. 4j. The second term *congestion* is a surrogate of the total time spent in the system; in each time step $t, A_{1i}(t)$ and $D_i(t)$ represents the number of vehicles (with destination *i*) that has entered and left the network, respectively. (In other words, $A_{12}(t) + A_{13}(t) - D_2(t) - D_3(t) = n_1(t) + n_2(t) + n_3(t)$.) The last term, *schedule deviation*, indicates the shift from the initial work schedule, i.e., $\widetilde{W}_i(t)$, to the optimal work schedule, i.e., $W_i(t)$. The decision variables include the travel times to each destination region, i.e. τ^*_{12}, τ^*_{13}, that we expect to observe and remain approximately constant under equilibrium conditions. Additionally, the values of the optimal wished curves, $W_2 = \{W_2(t) \mid 0 \leqslant t \leqslant t_f\}$ and $W_3 = \{W_3(t) \mid 0 \leqslant t \leqslant t_f\}$, are to be identified as part of the optimisation program. Note that t_f denotes the final time of process.

The constraints presented in Eq. 4b–4k arise from the illustration shown in Fig. 3 and the key principles outlined in Section 3.1 and Section 3.2. They represent the accumulation-based bathtub model (i.e., system dynamics) and the user equilibrium conditions in the bicentric network. Eq. 4b guarantees that all the travel demand is served between time 0 and t_f. Eq. 4c simply states that the wished curves and the departure curves are equal to each other, which implies there is no schedule cost in the optimal scenario that we aim for (i.e., approximated UE). Eq. 4d relates the derivative of the wished curve (or the departure curve) in the destination Region $i \in \{2, 3\}$ to the MFD function G_i and the accumulation of vehicles in Region i, n_i. The accumulation n_i is related to the departure curve from Region 1 to destination Region *i*, as presented by Eq. 4e. Similarly, the total outflow from Region 1, i.e., $\sum_{i \in \{2,3\}} \dot{D}_{1i}$ is equal to the MFD function G_1 and the accumulation n_1, see Eq. 4f. Obviously, as Eq. 4g shows, the resulting accumulation n_1 is the sum of partial accumulations in Region 1 with different destinations, i.e. $n_1(t) = \sum_{i\in\{2,3\}} n_{1i}(t)$, and as Eq. 4h shows, it is split in proportion to the outflow or transfer flows to destination regions. Eq. 4i relates the arrival curve A_{1i} to the congestion dynamics considering the departure curve D_{1i} and the partial accumulation n_{1i}. On the other hand, Eq. 4j defines the (expected) equilibrium arrival curve, i.e., $\widetilde{A}_{1i}$, as a curve parallel to the wished curve, i.e., W_i, with a time lag of τ^*_{1i}. In other words, $\widetilde{A}_{1i}$, as it is parallel to the wished curve W_i or the departure curve D_i with a time lag of τ^*_{1i}, allows us to define the equilibrium conditions where every traveller (with destination i) has approximately the same travel time τ^*_{1i} and has no earliness/lateness cost, i.e., $A_{1i} \approx \widetilde{A}_{1i}$. Lastly, Eq. 4k requires the positivity of the states of the system.

$$\underset{\boldsymbol{W_2}, \boldsymbol{W_3}, \tau^*_{12}, \tau^*_{13}}{\text{minimize}} \quad \sum_{i\in\{2,3\}} \int_0^{t_f} \left[\alpha\left(A_{1i}(t) - \widetilde{A}_{1i}(t)\right)^2 + \beta(A_{1i}(t) - D_i(t))^2 + \gamma\left(W_i(t) - \widetilde{W}_i(t)\right)^2 \right] \mathrm{d}t \tag{4a}$$

$$\text{subject to} \quad W_i(0) = 0, \quad W_i(t_f) = \widetilde{W}_i(t_f) \quad i \in \{2,3\} \tag{4b}$$

$$\text{for } t \in [0, t_f]: \quad W_i(t) = D_i(t) \quad i \in \{2,3\} \tag{4c}$$

$$\dot{W}_i(t) = G_i(n_i(t)) \quad i \in \{2,3\} \tag{4d}$$

$$D_{1i}(t) = D_i(t) + n_i(t) \quad i \in \{2,3\} \tag{4e}$$

$$G_1(n_1(t)) = \sum_{i\in\{2,3\}} \dot{D}_{1i}(t) \tag{4f}$$

$$n_1(t) = \sum_{i\in\{2,3\}} n_{1i}(t) \tag{4g}$$

$$\frac{n_{12}(t)}{n_{13}(t)} = \frac{\dot{D}_{12}(t)}{\dot{D}_{13}(t)} \tag{4h}$$

$$A_{1i}(t) = D_{1i}(t) + n_{1i}(t) \quad i \in \{2,3\} \tag{4i}$$

$$\widetilde{A}_{1i}(t) = W_i(t + \tau^*_{1i}) \quad i \in \{2,3\} \tag{4j}$$

$$n_{1i}(t) \geqslant 0, \quad n_i(t) \geqslant 0 \quad i \in \{2,3\}. \tag{4k}$$

The above formulation defines the optimal wished curve values $\boldsymbol{W_2}$ and $\boldsymbol{W_3}$ in continuous format. To solve this optimisation program, as Eq. 5 shows, we define the optimal wished curve as a generalised logistic function with 5 parameters to identify, i.e., ω_i. Briefly, generalised logistic functions are proposed as an extension of logistic or sigmoid functions, with the capability of producing more flexible S-shaped curves, including asymmetric curves. We choose to represent the optimal wished curves as generalised logistic functions due to its flexibility in creating asymmetric curves and its parametric nature, which allows an elegant optimisation structure. Other functional forms of the wished curve can be readily integrated in the proposed optimization. On the other hand, Eq. 6 defines the equilibrium arrival curve, i.e., $\widetilde{A}_{1i}$, as a generalised logistic function with the same parameters ω_i but with a time lag of τ^*_{1i}. In other words, $\widetilde{A}_{1i}$ is parallel to the wished curve W_i or the departure curve D_i with a time lag of τ^*_{1i}. By defining the optimal wished curves as generalised logistic functions, we replace the decision variables $\boldsymbol{W_2}, \boldsymbol{W_3}$ with the parameters of the logistic function $\omega_2^1, \cdots, \omega_2^5$ and $\omega_3^1, \cdots, \omega_3^5$, and allow the travel times τ^*_{12}, τ^*_{13} to be continuous variables, which makes the optimisation structure tractable. Additionally, as the optimal wished curve W_i follows a generalised logistic function, its derivative in Eq. 4d can be analytically computed; however, the derivatives $\dot{D}_{1i}$ require a numerical differentiation. As we build the system dynamics with fine time intervals, we note that the numerical differentiation produces accurate results. It is worth mentioning that the initial wished curve $\widetilde{W}_i$ is an input to the optimisation problem, and does not have to follow a generalised logistic function.

$$W_i(t) = \omega_i^1 + \frac{\omega_i^2 - \omega_i^1}{\left(1 + \omega_i^3 \cdot \exp(-\omega_i^4 \cdot t)\right)^{1/\omega_i^5}} \quad i \in \{2,3\} \tag{5}$$

$$\widetilde{A}_{1i}(t) = \omega_i^1 + \frac{\omega_i^2 - \omega_i^1}{\left(1 + \omega_i^3 \cdot \exp(-\omega_i^4 \cdot (t + \tau^*_{1i}))\right)^{1/\omega_i^5}} \quad i \in \{2,3\} \tag{6}$$

The overarching objective of the optimal work schedule problem is to minimize the traffic congestion in the network (the second term in Eq. 4a) under near-equilibrium conditions (the first term in Eq. 4a) with minimal changes in the initial work schedules (the third term in Eq. 4a). These objectives may have a conflicting nature; in order to minimise the traffic congestion in the system, one needs to make significant changes in the work schedules, and possibly create uniformly distributed work schedules across the peak hours. Nevertheless, applying such changes through employer taxation and other forms would not be easy, and cause significant compliance issues. On the other hand, it is obvious that with no changes in the initial work schedules, traffic conditions would not improve. Additionally, the changes in the work schedules will change departure times, which realistically captures the reaction of travellers under user equilibrium assumptions. In other words, the optimal work schedule scenario should take account of travellers' reaction when optimising the congestion cost with minimal schedule changes. Intuitively, these objectives are often conflicting; hence, we choose to formulate the optimal work schedule problem as a multi-objective optimisation problem. Considering the nature of the problem, it is clear that no single solution exists that simultaneously optimizes each objective. Therefore, with assigning different weight parameters to each objective/term in the objective function, we find a representative set of Pareto optimal solutions. While it is difficult to argue that one solution is better than others in the Pareto frontier, our aim is to quantify the trade-offs in satisfying the different objectives, and identify a compromising solution that accounts for multiple aspects of the problem.

The optimization problem is a non-convex nonlinear program (NLP) due to the nonlinear nature of MFD (a third-order polynomial function is adopted in this study) and the resulting congestion propagation dynamics as presented in Eq. 4b–4 k. Note that the congestion propagation dynamics serve as the constraints to the optimization problem and define a nonlinear and non-convex solution space for the optimization problem, see for instance Eq. 4h. This makes the whole optimization problem non-convex and nonlinear. The resulting constrained NLP includes 12 decision variables; the parameters of the logistic function $\omega_2^1, \cdots, \omega_2^5$ and $\omega_3^1, \cdots, \omega_3^5$, and the equilibrium travel times τ_{12}^*, τ_{13}^*. The resulting NLP can be efficiently solved via, e.g., sequential quadratic programming or interior point solvers (see Diehl et al. (2006) for details). In this study, software implementation of the optimization is done using the CasADi toolbox (Andersson et al., 2018) in which the NLPs are solved by the interior point solver IPOPT (Wächter and Biegler, 2006). A tolerance level of 10^{-8} and 3000 maximum iterations has been used in all runs to identify the optimal solution.

4.2. Employer-level optimisation

In Section 4.1, we described the upper level of the optimal work schedules problem in which the departure curves (or wished curves $W_2(t)$ and $W_3(t)$) are optimized. Those curves identify the outflow rate from each city-center (Region 2 or 3) that is the work start times of multiple firms aggregated at the level of region. To complete the overall optimal work schedule problem, we develop an optimization program that decomposes the optimal work schedules to work start times of different firms in the region. The following is the lower level (employer-level) of the staggered work schedule optimization problem that is considered for each region independently. Thus for the sake of brevity, the region index is omitted.

Let us assume $\mathcal{K}$ denotes the set of firms in the region indexed by k. Further, the decision variables are the incentivized work start periods of the firms, i.e. $[t_k^s, t_k^e]$, the beginning of the work start period and the end time of the work start period of firm k respectively.

$$\underset{t_k^s, t_k^e}{\text{minimize}} \quad \sum_{k \in \mathcal{K}} \left[|t_k^s - \tilde{t}_k^s| + |t_k^e - \tilde{t}_k^e| \right] + \delta \int_0^{t_f} [|W(t) - W^*(t)|] \mathrm{d}t \tag{7a}$$

subject to

$$0 \leqslant t_k^s < t_k^e \leqslant t_f \quad k \in \mathcal{K} \tag{7b}$$

$$\Delta_k \leqslant t_k^e - t_k^s \quad k \in \mathcal{K} \tag{7c}$$

$$W^*(t) = \mathcal{F}([t_k^s, t_k^e]) \quad k \in \mathcal{K}. \tag{7d}$$

Eq. 7a represents the objective function of the lower level optimization program. The first term in Eq. 7a measures the deviations between the incentivized work start periods and the initial work start periods of the firms, $[\tilde{t}_k^s, \tilde{t}_k^e]$. The second term weighted by δ measures the difference between the optimal work schedule of the region, $W(t)$, and the incentivized work schedule of the region aggregated from individual work start period of firms. Note that $W(t)$ is an input in Eq. 7a while it is the output of the upper level multi-objective optimization program in Eq. 4a. The incentivized beginning and end times of the work start period of firm k, i.e. respectively t_k^s and t_k^e, should be between time 0 and the final time t_f. Also, the incentivized beginning of the work start period of firm k should be before the end time of the work start period. These constraints are considered in Eq. 7b. To accommodate a work start period for firms rather than a single point of time, $0 < \Delta_k$ is considered in Eq. 7c to guarantee a minimum duration for the work start period of firm k (e.g. 15 min). Lastly. the aggregated incentivized work schedule of the region, $W^*(t)$, is simply the superposition of work start periods of all firms that is represented with function $\mathcal{F}$. Note that $\mathcal{F}$ also takes into consideration the number of employees of all the firms.

The employer-level optimization program, Eq. 7a, is nonlinear and constrained with a non-empty feasibility set. Thus, the solution existence can be verified but the uniqueness cannot be established. Note that, the non-uniqueness of Eq. 7a solution does not pose a challenge in the overall proposed method since the wished curve optimisation program in Eq. 4a, does not depend on the solution of Eq. 7a. The optimization program 7a comprises of two times of the number of firms decision variables; the incentivized work start periods of the firms. The optimization program can be efficiently solved using different numerical solvers. In this paper, we resort to the interior point algorithm implemented in Matlab. We run 40 instances of the (local) optimization with random initial points with the tolerance on the constraint violation equal to 10^{-6}, the maximum number of iterations equal to 1000, the maximum number of function evaluations set to 3000, the termination tolerance on the first-order optimality as 10^{-6}, and the termination tolerance on the decision variables set as 10^{-10}. The details of the results of the employer-level work start time optimization is presented in Section 5.3.

5. Numerical experiments

This section is structured as follows. First, we provide the numerical settings for the proposed case study. Second, we discuss the results of the top-level optimisation problem of the regional work schedules. Third, we present the results of the employer-level schedule design problem. Finally, we test the proposed strategy using the trip-based MFD model, and provide a comparison to the baseline scenario.

5.1. Numerical settings

The numerical case study is based on the bicentric network presented in Fig. 2. In numerical experiments, we use two MFD-based traffic models; the accumulation-based and the trip-based MFD models, as outlined in Fig. 1. The accumulation-based model is the basis of the multi-objective schedule optimisation problem to identify the optimal solution for the work schedules. The trip-based MFD model together with the iterative UE approach is used as the testbed (i.e., the *plant* representing reality) for two purposes; (i) to identify the baseline scenario[1] where the user equilibrium conditions should be established with a generalised travel cost including both the travel time and the earliness/lateness cost, and (ii) to assess the proposed optimal work schedule scenario considering both the quality of the equilibrium approximation and the resulting traffic conditions. Note that while a numerical approximation of the analytical equilibrium solution exists for single region networks (Amirgholy and Gao, 2017), such solutions are not available for the multi-regional networks, e.g., the bicentric network under study. Therefore, we resort to the trip-based model and aim to establish user equilibrium conditions through an iterative framework, where we employ a day-to-day departure time assignment model (Yildirimoglu and Ramezani, 2019). The pseudo algorithm of the trip-based MFD model for the bicentric network is provided in Appendix A. For more details on the implemented trip-based MFD model see (Yildirimoglu and Ramezani, 2019; Li et al., 2021).

Tables 1 and 2 present the parameters for the accumulation-based and the trip-based model, respectively. These two models rely on equivalent representations of the MFD function, but have inherently different approaches to model traffic congestion dynamics. While the travellers are subject to varying trip lengths and earliness/lateness coefficients in the trip-based MFD model, the schedule optimization model employs the accumulation-based MFD model where the average trip length is constant. Note that without loss of generality the same production or speed MFD functions and the average trip lengths are used in all three regions. The outflow MFD function, which is considered in the multi-objective formulation, is $G(n) = P(n)/\bar{l}$.

Although the optimisation problem (Eq. 4b–4 k) is presented in a continuous format, we discretize the problem to solve it with digital computers. Time step in the accumulation-based model and so the multi-objective optimisation formulation is $\Delta t = 10$ s. Additionally, for each $\Delta t = 10$ s, we apply Runge Kutta (RK4) method using a step size of 1 s. The trip-based MFD model is inherently event-based with no constant time step.

Table 1
Accumulation-based MFD model parameters.

Accumulation-based model	Specification
Production function	$P(n) = a \cdot n^3 + b \cdot n^2 + c \cdot n$ [veh·m/s]
Outflow MFD	$G(n) = P(n)/\bar{l}$
Parameters	$a = 9.98 \cdot 10^{-8}, b = -0.002, c = 9.78$
Parameters	$\bar{l} = 2300[m], n^{\text{jam}} = 10^4$ [veh], $n^{\text{cr}} = n^{\text{jam}}/3$ [veh]

Table 2
Trip-based MFD model parameters. $\mathcal{N}$ and $\mathcal{U}$ represent the Gaussian and uniform distributions respectively.

Trip-based model	Specification
Speed function	$V(n) = P(n)/n$ [m/s]
Trip length	$\bar{l} + \mathcal{N}\left(0, (0.1 \cdot \bar{l})^2\right)$
Earliness parameter	$\mathcal{U}(0.3, 0.7)$
Lateness parameter	$\mathcal{U}(2.5, 5.5)$

[1] The baseline scenario indicates the traffic conditions that result from the initial wished curve. This scenario is presumably congested, and involves schedule costs that allow for temporal equilibrium conditions to hold.

5.2. Multi-objective optimisation results

To determine the Pareto frontier resulting from the multi-objective formulation in Eq. 4a, we solve the optimisation problem with possible combinations of the following weight parameters; $\alpha = \{1000\}, \beta = \{0.25, 0.50, 0.75, 1.00, 2.50, 5.00, 10.00, 25.00, 50.00\}$, and $\gamma = \{0.25, 0.50, 0.75, 1.00, 2.50, 5.00, 10.00, 25.00, 50.00\}$. We deliberately assign a high value and do not change the value of α as this term indicates the penalty for constraint violation in relation to the equilibrium approximation.

Fig. 4 presents the optimal solutions in the 3 hyperplanes (i.e., *congestion-schedule deviation, equilibrium-schedule deviation,* and *congestion-equilibrium*) color-coded with respect to the value of β and γ, while $\alpha = 1000$ in all the tests. In all the 3 hyperplanes, the optimal solutions are sensitive to the value of γ (i.e., the weight for the *schedule deviation* term) as evidenced by the grouping of results with respect to the value of γ in Figs. 4(a), (c), and (e). However, the value of β (i.e., the weight for the *congestion* term) plays a lesser role in the optimal value of the three objectives. We note that the *schedule deviation* term is inversely correlated with the *congestion* and the *equilibrium* terms (see Figs. 4(a) and (c)). This makes intuitive sense; the more we shift from or spread the initial schedule, the further we reduce the congestion in the system. On the other hand, there is a positive correlation between the *congestion* and the *equilibrium* terms, because the more congested, the more difficult it is to satisfy equilibrium conditions where every traveller presumably has the same travel time.

Fig. 4(a) shows that the congestion term reaches its minimum for $\gamma = \{0.25, 0.50, 0.75\}$, where it is less than half of its maximum value at $\gamma = 50.00$. In fact, the congestion term becomes insensitive to the the changes in γ below 0.75; in other words, while the *schedule deviation* values keep rising with decreasing values of γ below 0.75, the *congestion* cost remains more or less constant. This implies that making further changes in work schedules below $\gamma = 0.75$ has little to no impact on the *congestion* cost in the system. Additionally, the scenarios with $\gamma = \{0.25, 0.50, 0.75\}$ are accompanied with high *schedule deviation* values and therefore indicate a significant shift in the work schedules, which may not be easy to implement. At the other end of the spectrum, we note that the *schedule deviation* term plateaus with the increasing *congestion* cost and reach very low values, which represents a scenario very similar to the initial conditions. For $\gamma = 1.00$ and $\gamma = 2.50$, we note that the *schedule deviation* is significantly low compared with its maximum value at $\gamma = 0.25$, and that there is significant improvement in the associated congestion cost. While it is hard to argue that one of the optimal solutions in the Pareto frontier is better than others, the optimal solutions at $\gamma = 1.00$ and $\gamma = 2.50$ present a good compromising solution where the *schedule deviation* from the initial scenario is moderate and the *congestion* is significantly curbed.

The pattern in Fig. 4(c) is very similar to the one in Fig. 4(a); from the *equilibrium* perspective, the difference between the solution at $\gamma = 0.25$ and the solution at $\gamma = 1.00$ or $\gamma = 2.50$ is quite minimal, compared with the higher *equilibrium* terms that we observe at $\gamma = 50.00$. We also note that the *equilibrium* term differs from the *schedule deviation* and *congestion* values by two orders of magnitude, as intended by the high value of α=1000, see the axis ranges in Fig. 4. Additionally, even with the low values of $\beta = 0.25$ and $\gamma = 0.25$, *equilibrium* term does not vanish, albeit becoming extremely small. This confirms our initial hypothesis that satisfying exact equilibrium conditions in the proposed schedule design procedure is quite difficult, and shows the value in the adopted constraint violation approach.

Fig. 4(e), as discussed earlier, depicts a positive correlation between the *congestion* and the *equilibrium* terms; in other words, these two objectives are not conflicting. Nevertheless, the changes in the *equilibrium* values are quite minimal compared with the changes in the *congestion* values. Again, the solutions with $\gamma = 1.00$ and $\gamma = 2.50$ indicate a good compromising scenario for both the *congestion* and the *equilibrium* terms. Considering the findings from the Pareto frontier analysis, we conclude that the optimal solution corresponding to $\{\alpha, \beta, \gamma\} = \{1000, 2.50, 2.50\}$ is an adequate scenario that can be further tested to assess the full impact of the proposed method.

Fig. 5(a) and (b) presents the cumulative arrival, departure, and wished curves corresponding to the optimal solution with $\{\alpha, \beta, \gamma\} = \{1000, 2.50, 2.50\}$. Note that the total number of vehicles going to Regions 2 and 3 in the peak period are not the same. The figures also include the initial wished curve $\widetilde{W}_i$ of Region *i* and the equilibrium arrival curve $\widetilde{A}_{1i}$, i.e., the arrival curve that satisfies temporal equilibrium conditions. As the temporal equilibrium conditions dictate the same travel time for all users of the same origin-destination pair, $\widetilde{A}_{1i}$ must be parallel to the departure curve (or the wished curve, as we assume no schedule cost exists) with a time lag of τ_i which itself is a decision variable in the multi-objective optimisation problem, see Eq. 4a. Note that, in both Fig. 5(a) and (b), the equilibrium arrival curve $\widetilde{A}_{1i}$ is very similar to the actual arrival curve A_{1i}, which implies that the resulting optimal solution adequately captures the equilibrium conditions and provides approximately the same travel time to all users. This is an important result emphasizing the applicability and the stability of the proposed scheme; the changes in the work schedule inherently change the departure time patterns, but the resulting patterns align with the temporal equilibrium assumptions.

Note that traffic conditions are not constant throughout the peak period; Fig. 5(c) shows the changing accumulation values in the bicentric urban network. However, as the congestion in the network is dramatically curbed as a result of the schedule optimisation process (the critical accumulation is about 3300 [veh]), the change in the experienced speed in the peak period is minimal, which allows a decent approximation of the user equilibrium conditions, in other words a good agreement between $\widetilde{A}_{1i}$ and A_{1i}. We also observe that the shift from the initial work schedules (i.e., the difference between the initial wished curve $\widetilde{W}_i$ and the optimal wished curve W_i) is significant but not excessive in both regions. Fig. 5(d) allows a comparison of the wished curves in Regions 2 and 3. We note that, although the total number of vehicles and the initial schedule distribution is quite different, the proposed optimization program modifies both curves in a similar manner and yields minimal changes in the schedules particularly for those wanting to arrive in the middle of the peak period, see e.g., the difference between W_2 and $\widetilde{W}_2$ from $t = 80$ min to $t = 120$ min, and the difference between W_3 and $\widetilde{W}_3$ from $t = 60$ min to $t = 100$ min.

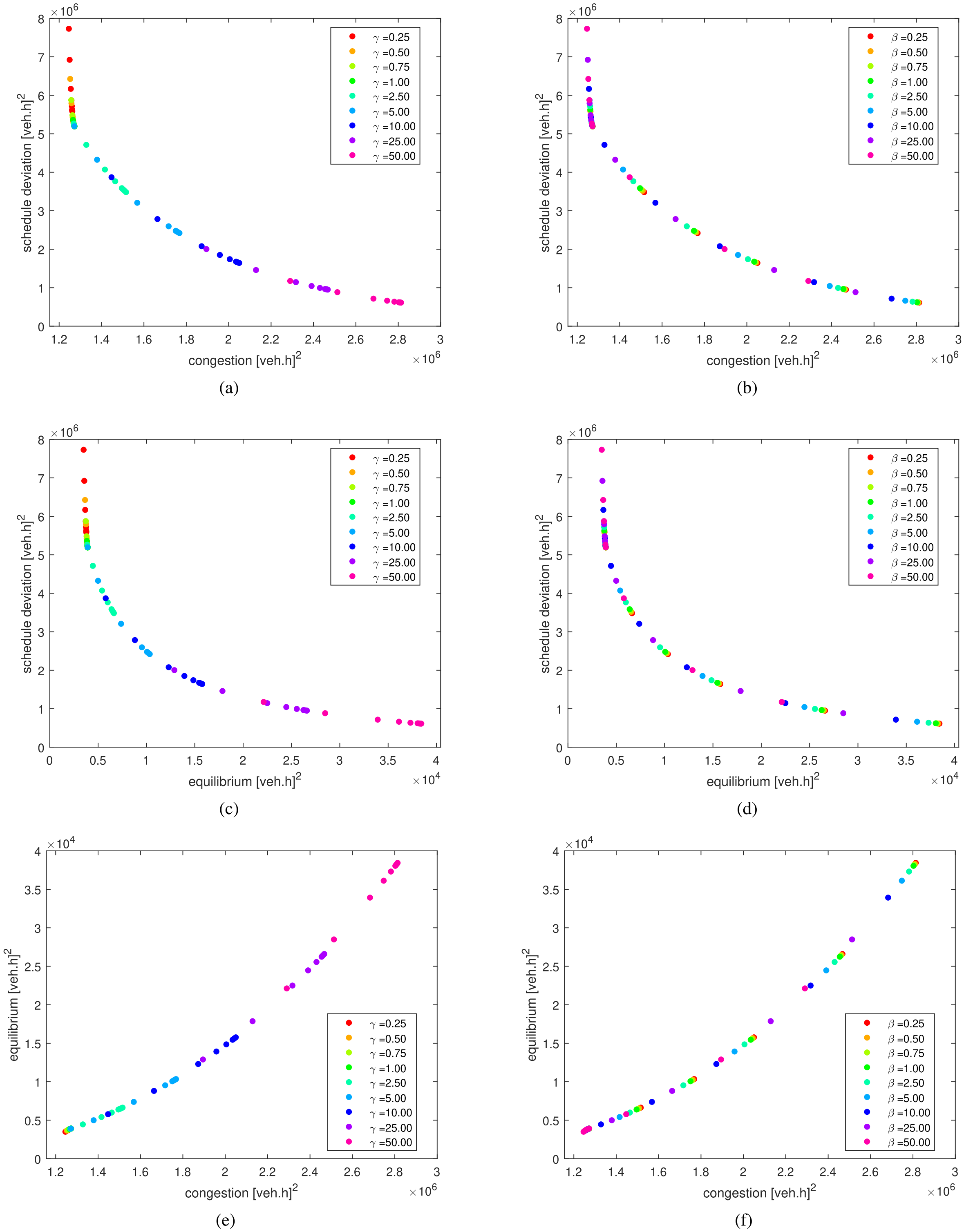

Fig. 4. Pareto optimal solutions; equilibrium, congestion, and schedule deviation refer to the three terms in Eq. 4a weighted by α, β, γ, respectively.

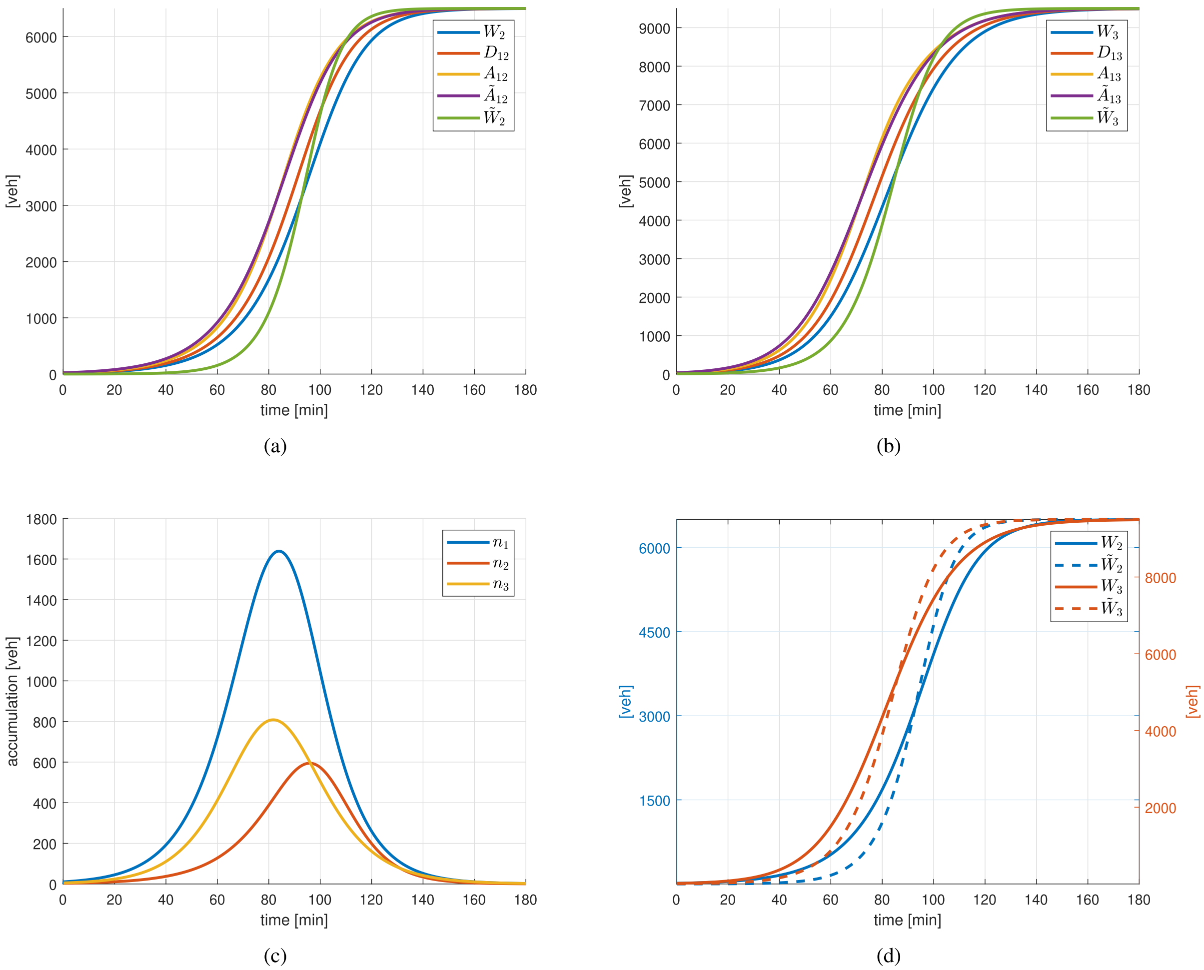

Fig. 5. Results from the multi-objective optimisation; (a) cumulative arrival, departure, and wished curves for vehicles going from Region 1 to 2, (b) cumulative arrival, departure, and wished curves for vehicles going from Region 1 to 3, (c) accumulations in the three regions, (d) initial and optimal wished curves for Regions 2 and 3.

5.3. Results of employer-level optimization

To fully evaluate the performance of the proposed staggered work scheduling optimization, the optimized wished curves of Regions 2 and 3, i.e. W_2 and W_3, are used in the employer-level optimization problem as in Eq. 7a. We consider ten different firms in each region and allocate them with initial work start times such that the aggregated regional wished curves represent the initial wished curves as in the dashed lines in Fig. 5(d). Each firm in Regions 2 and 3 has 650 and 950 employees respectively and we assume employees of each firm arrive to their work locations uniformly in time within the work start duration of the firm. We select $\delta = 1$ in Eq. 7a and $\Delta_k = 15\text{min}$ in Eq. 7c for $k = 1, \cdots, 10$.

Fig. 6 displays the results of the employer-level optimization. The incentivized work start times of each firm in Region 2 and Region 3 are displayed in Fig. 6(a) and (b) respectively. The aggregated superposition of the incentivized employers work start times are depicted in Fig. 6(b) and (d) for Regions 2 and 3 where a close match between incentivized wished arrival curves, i.e. W_2^* and W_3^*, and the optimal wished arrival curves from the upper-level optimization, i.e. W_2 and W_3, are apparent. This highlights that the proposed two-level staggered work schedule optimization can be implemented by incentivizing/regulating individual employers in each region to mitigate the network congestion while each traveller react to the incentivized work start times selfishly.

5.4. Baseline scenario and performance tests with the trip-based MFD model

To scrutinize the performance and validity of the proposed staggered work schedule optimization and the underlying accumulation-based MFD model, we input the optimized arrival curves to the network, A_{12} and A_{13}, that are the output of optimization Eq. 4a to the trip-based MFD testbed. The dynamics of congestion in this testbed are governed by the speed MFD. This testbed is more disaggregated than the accumulation-based MFD model such that each vehicle is modeled individually by considering its unique

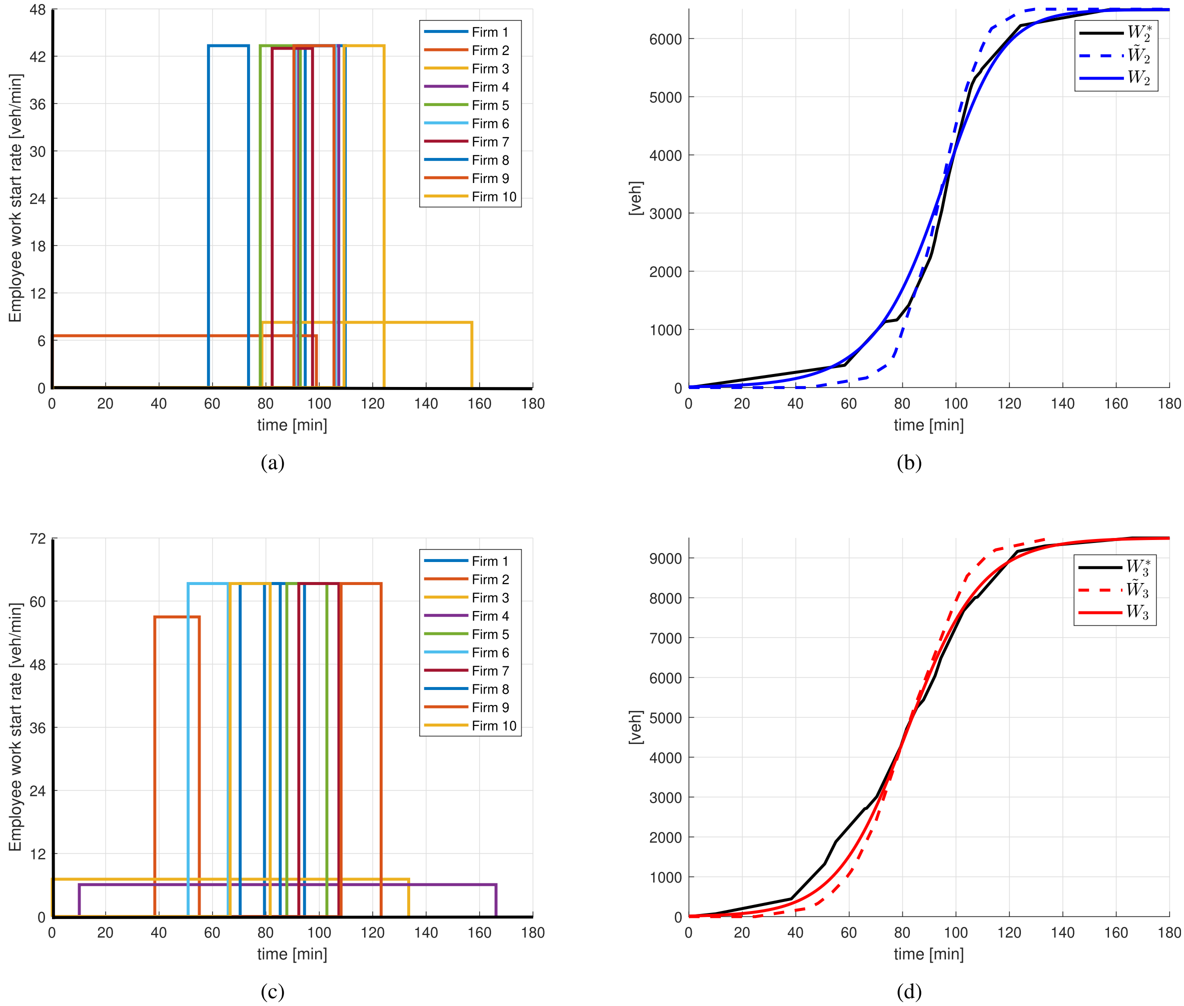

Fig. 6. Results of employer-level work start times; (a) incentivized work start times of firms in Region 2; (b) the incentivized, optimal, and initial cumulative wished curves of Region 2; (c) incentivized work start times of firms in Region 3; and (d) the incentivized, optimal, and initial cumulative wished curves of Region 3.

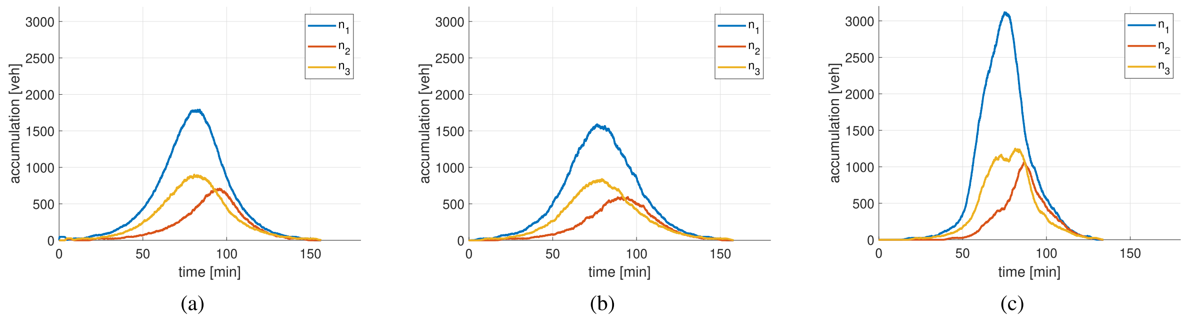

Fig. 7. The accumulations of Regions 1,2, and 3 in trip-based MFD testbed. (a) with the optimal arrival curves; (b) with the optimal wished curves; (c) baseline scenario with the initial wished curves. Results in (a) and (b) are consistent with each other demonstrating that the iterative assignment approach is valid. Consistency of (a) with the outcomes of the optimization based on accumulation-based MFD modeling as in Fig. 5(c) validates the overall accuracy of the optimal-staggered work schedule method.

arrival time, transfer time, departure time, trip length, and earliness and lateness parameters. Accordingly, travel time and schedule cost of each traveller under user equilibrium and schedule optimization scenarios can be estimated. The results of this test, i.e. accumulation of Regions 1,2, and 3, are depicted in Fig. 7(a). Comparison with the prediction of the proposed staggered work schedule optimization program in Fig. 5(c), demonstrates close similarity of accumulation of the three regions.

Moreover, we used the optimal wished curves, W_2 and W_3, and applied them in the day-to-day assignment model to compute the arrival time to the network (that is the departure time from the residential places). That is, commuters change their arrival time every day based on their travel times and schedule costs in previous days until they reach equilibrium conditions where they do not have an incentive to change their departure time. This requires an iterative use of the trip-based MFD model. Further details of the iterative procedure can be found in Yildirimoglu and Ramezani (2019). The resulting accumulations in the three regions under equilibrium conditions are presented in Fig. 7(b). The consistency between the accumulations of the three regions with forward-implementation of the trip-based MFD model, Fig. 7(a), and the iterative day-to-day implementation of the trip-based MFD model, Fig. 7(b), demonstrates that the proposed optimal staggered work schedule optimization can result in near-equilibrium temporal conditions. The results of the baseline scenario corresponding to the initial wished curves under equilibrium conditions are presented in Fig. 7(c). Evidently, all the three regions experience higher accumulation of vehicles. The network total travel time in baseline scenario is 10.18×10^6 [veh.sec] and the total schedule cost is 28.54×10^5 [veh.sec]. The proposed optimal staggered work schedule results in the total travel time and total schedule cost as 8.79×10^6 [veh.sec] and 7.22×10^5 [veh.sec] respectively. This is 14% and 75% decrease in total travel time and total schedule cost respectively.

6. Conclusion

In this paper, we have proposed a method to design optimal work schedules in bicentric cities. While it is clear that traffic congestion can be alleviated by spreading the peak demand distribution, a rigorous method is needed in the design of optimal work schedules (i) to minimise traffic congestion, (ii) to produce a desirable outcome that is consistent with temporal equilibrium conditions of commuters arrival time choice, and (iii) incurring minimal changes in the initial work schedules of employers. This paper fills this gap, and develops a multi-objective optimisation problem that builds on the accumulation-based bathtub MFD model. The proposed framework also accounts for multiple employers with various schedules, and decomposes the regional aggregated work schedule distributions into starting times for separate businesses through a lower-level optimisation problem. The results from a bicentric network are promising; traffic congestion can be significantly curbed with little changes in the initial work schedules while satisfying temporal equilibrium conditions.

A future research direction might be to incorporate freeway links into the modelling framework; downtown areas in most cities are connected to the peripheral network with critical freeway infrastructure which often gets congested in the peak hours. While MFD adequately captures the congestion propagation in the urban areas, a disaggregated link-level traffic modeling approach is needed for freeway links. This requires the integration of the existing aggregated modeling framework and a dedicated freeway model in a hybrid modelling environment (Haddad et al., 2013; Han et al., 2020). Note that temporal equilibrium conditions in this framework should account for both link-level and region-level congestion characteristics. Considering other modes of travel specifically public transport options in forms of buses and trains would add more realistic dimensions into the traffic modeling (Loder et al., 2017). Additionally, the proposed model can be tested with a microscopic simulation model to further consider the modeling mismatch between our aggregated and low complexity model (i.e., accumulation-based bathtub model) and the fine-grained traffic simulation. Finally, it is evident that to adopt and sustain staggered work schedules policies and to achieve their ultimate goal, which is congestion mitigation, cities have to incentivize employers and/or employees. This is a key challenge and a crucial aspect of the staggered work schedules problem that is a priority for future research.

CRediT authorship contribution statement

Mehmet Yildirimoglu: Conceptualization, Data curation, Formal analysis, Methodology, Validation, Visualization, Writing – original draft, and Writing – review & editing. **Mohsen Ramezani:** Conceptualization, Data curation, Formal analysis, Methodology, Validation, Visualization, Writing – original draft, and Writing – review & editing. **Mahyar Amirgholy:** Conceptualization, Formal analysis, Methodology, Writing – original draft, and Writing – review & editing.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Appendix A. Trip-based MFD algorithm for the bicentric network

Algorithm 1. Trip-based MFD pseudo-code

Initialize each traveller (i): $\{t_i^{\text{arr}}, T_i^{\text{w}}, d_i, l_i^1, l_i^{d_i}$, scheduling penalties$\}$
```
%arrival time of traveller to Region 1; wished departure time of traveller from
    destination region; destination region (2 or 3); traveller trip length in
    Region 1; traveller trip length in destination region; scheduling penalties
```
Initialize event_list = []
Initialize $[n_{12}, n_{13}, n_2, n_3] = [0,0,0,0]$ `%vehicle accumulation`
Initialize $j = 0$ `%event counter`
Initialize $t_j = t_{\text{init}}$
current_speed = $[V_1(n_1), V_2(n_2), V_3(n_3)]$
for All travellers (i) **do**
  event_list ← t_i^{arr}, the arrival time to Region 1
  event_list ← t_i^{tra}, the potential transfer time from Region 1 to the destination region considering constant $V_1(n_1)$
  event_list ← t_i^{dep}, the potential departure time from the destination region considering constant $V_1(n_1)$ and $V_{d_i}(n_{d_i})$
end for
Sort event_list
while Items in event_list **do**
  $j \leftarrow j+1$
  Determine the next event closest to t_{j-1} and let its corresponding time as t_j
  $l_i^1 \leftarrow l_i^1 - V_1(n_1)\cdot(t_j - t_{j-1}) \quad \forall i$ in Region 1 `%update the remaining trip length in Region 1`
  $l_i^{d_i} \leftarrow l_i^{d_i} - V_{d_i}(n_{d_i})\cdot(t_j - t_{j-1}) \quad \forall i$ in Region d_i `%update the remaining trip length in destination region`
  if the first closest event is a vehicle arrival **then**
    if $d_i == 2$ **then**
      $n_{12} \leftarrow n_{12} + 1$
    else
      $n_{13} \leftarrow n_{13} + 1$
    end if
  end if
  if the first closest event is a vehicle transfer **then**
    if $d_i == 2$ **then**
      $n_{12} \leftarrow n_{12} - 1; n_2 \leftarrow n_2 + 1$
    else
      $n_{13} \leftarrow n_{13} - 1; n_3 \leftarrow n_3 + 1$
    end if
  end if
  if the first closest event is a vehicle departure **then**
    if $d_i == 2$ **then**
      $n_2 \leftarrow n_2 - 1$
    else
      $n_3 \leftarrow n_3 - 1$
    end if
    output ← $[T_i^{\text{exp}}, \text{schedulingpenalties} \times (T_i^{\text{w}} - T_i^{\text{exp}})]$ `%the experienced travel time and schedule cost of vehicle` i
  end if
  Remove the first closest event from the event_list
  Update $[V_1(n_1), V_2(n_2), V_3(n_3)]$
  Update the potential transfer and departure times of all vehicles in event_list considering new constant $[V_1(n_1), V_2(n_2), V_3(n_3)]$
end while

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