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Incentivizing shared rides in e-hailing markets: Dynamic discounting☆

Guipeng Jiao, Mohsen Ramezani *

The University of Sydney, School of Civil Engineering, Sydney, Australia

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ABSTRACT

Ridesourcing service provided by transportation network companies (TNCs) has been shown by various studies to deteriorate traffic conditions as they increase the number of unoccupied vehicles on the road. Ridesharing is an alternate service where passengers with similar itineraries may be pooled together in a combined trip, which could counteract some of the negative externalities of ridesourcing. Passengers are generally compensated with a price discount for their inconveniences experienced in shared trips. We therefore propose a dynamic discount pricing strategy for the platform to incentivize ridesharing. This study explicitly considers the passengers to be cost and service quality sensitive, impatient, and have a choice to decline services offered by the platform. The strategy is integrated in the matching algorithm which maximizes the platform's profit with given batches of waiting passengers and idle vehicles, and strategically offers shared trips to *selected* passengers with varying price discounts. We show that the adoption of the dynamic discount pricing strategy creates substantial economic benefit for the platform in both the short and long terms. The drivers are also shown to benefit from the strategy as they spend more time serving passengers. The strategy reduces the fleet size required to service the same number of passengers and is beneficial to all stakeholders.

1. Introduction

Transportation Network Companies (TNCs) such as Uber, Didi, and Lyft have emerged and grown explosively over the past decade. Those TNCs offer e-hailing platforms to provide point-to-point on-demand mobility services. The ability for the platforms to match passengers and drivers, and to implement dynamic pricing strategies provides them a competitive edge over the traditional taxis in the ride-hailing market. Ridesourcing services provided by TNCs can be considered a direct substitute to that by the taxis, where there is one destination per trip. However, it has been shown by various studies that ridesourcing has deteriorated traffic conditions as it increases the number of unoccupied vehicles on the road (e.g. Schaller, 2017; Beojone and Geroliminis, 2021), which subsequently lead to more pollution and congestion (Anair et al., 2020; Barrios et al., 2020; Ramezani and Nourinejad, 2018).

Ridesharing is another service also often provided by TNCs and is enabled by the real-time matching nature (Chen et al., 2021) of these e-hailing platforms, where passengers with similar itineraries may be pooled together in a combined trip. Though less popular among passengers (Xu et al., 2021; Brown, 2020), ridesharing could counteract some of the negative externalities of ridesourcing, as it might increase the number of passengers on board and enable a smaller fleet to service the same number of passenger requests (Alisoltani et al., 2021). The lower popularity of ridesharing can be attributed to the potentially longer waiting time and detour (Wang et al., 2021), and the inconvenience of sharing with a stranger (Irannezhad and Mahadevan, 2022). Therefore,

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* Corresponding author.
E-mail addresses: gjia5714@uni.sydney.edu.au (G. Jiao), mohsen.ramezani@sydney.edu.au (M. Ramezani).

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the passengers in shared trips are generally compensated with a price discount (Bahrami et al., 2022). Higher discount offered makes ridesharing more appealing to passengers but irrevocably lowers the profit generated per trip and vice versa. Ergo, the price discount is a crucial variable a platform has control over. In this paper, we formulate a dynamic ridesharing discount pricing strategy for a profit maximizing TNC and the ramifications of such strategy are evaluated.

Pricing in the ridesourcing market is studied in the literature extensively with the equilibrium approach. Wang et al. (2016) show the existence of equilibrium solutions for any pricing strategy, and identify a conflict between maximizing a platform's profit and improving the social welfare. Surge pricing or dynamic pricing is commonly used by TNCs, where prices react to imbalances between demand and supply. Considering price variation across time, Zha et al. (2017) investigate the impact of surge pricing and also present a trade-off, where the platform and the drivers enjoy higher earnings, while the customers may be at a loss. Cachon et al. (2017) compare several contractual forms that allow the platform to dynamically adjust the prices and/or wages. They find that surge pricing, where there is a constraint of fixed commission rates, achieves the near-optimal profit. Banerjee et al. (2015) on the other hand, using a queueing-theory economic model, show dynamic pricing strategy cannot exceed the throughput and revenue performance compared to the optimal static pricing policy. Comparing to adjusting prices temporally, spatial pricing is another dimension considered in the literature. For example, Zha et al. (2018) develop a discrete time geometric matching model and find platform and drivers are better off under revenue-maximizing spatial pricing. Guda and Subramanian (2019) also analyse the effects of spatial pricing strategies, so drivers can be made to move from a zone with excess supply, while offering drivers bonuses to move can achieve even higher profitability for the platform.

Fewer studies have explored pricing strategies in non-equilibrium settings. Notably, Nourinejad and Ramezani (2020) use a model predictive control framework and demonstrate that by relaxing the assumption that price is always higher than wage at all time, platform profit under dynamic pricing can be further improved. They argue that in practice, the market conditions may not remain untouched for sufficiently long periods to attain equilibrium. This paper takes the same stance. We consider a dynamic non-equilibrium model formulating the interactions among market participants.

In this study, we consider the platform offers customized (dynamic) discounts to shared trips, which are *individualized* towards passengers. Note that the strategy does not fall into the category of price discrimination for the following reasons, (i) it is a discount policy, and the original price structure of the service provided is not manipulated, (ii) the services provided to customers are not the same considering passengers experience different amount of detour and wait time which fundamentally alters the quality of the service, and (iii) the strategy does not take into consideration of the passengers' personal attributes, but only the spatial orientation of their origins and destinations.

The existing studies of ridesharing operations have diverse objectives, methodologies, and assumptions. Ke et al. (2020) develop an equilibrium model for the ridesharing market. They analyse the impact on the platform's profit and social welfare due to variations in trip fare and fleet size. They prove that the optimum solutions are located in the normal regime rather than the wild goose chase regime. Zhang and Nie (2021) also use an equilibrium model and find the ridesharing market system always has an equilibrium solution. They study the effects of pricing and regulations on platform profit and social welfare. Wei et al. (2020) model a multi-modal network with ridesharing services using a doubly dynamical approach. They find ridesharing reduces congestion if a congestion pricing scheme is implemented. Hosni et al. (2014) formulate a mixed integer program for the shared-taxi problem with the objective of profit maximization. They assume that each passenger specifies a pick-up and drop-off time window, as well as a maximum ride time, which must be satisfied for them to be in a shared ride. Qian et al. (2017) develop an integer linear programming problem and convert it into an equivalent graph problem for taxi group rides to maximize total saved travel miles. They assume that all passengers are willing to share their rides and they may be asked to meet at a designated location, and will be dropped at a place which comprises the needs of all people on board. Alonso-Mora et al. (2017) also introduce an integer linear program for the ridesharing problem to minimize the total sum of delays, and solve incrementally from a greedy assignment. Azadeh et al. (2022) present a model where the platform offers an assortment of alternative services for the passengers to choose from upon their arrival. With the goal of profit maximization, they show that the model benefits the platform and the customers. Fielbaum et al. (2021) investigate how to split the cost among passengers in shared trips. They consider the scenario as a non-cooperative game, and propose cost-sharing protocol that yields optimal solutions in three different equilibrium conditions. Jung et al. (2016) develop a hybrid-simulated annealing method for the shared-taxi dispatch algorithm for total passenger travel time minimization and system profit maximization, while maximum waiting time and detour assumptions are considered. The majority of the studies on ridesharing, in one form or another, assume all passengers are willing to share irrespective of service quality and/or price. However, in practice, passengers have individualized preferences regarding ridesharing which is unknown to the platform. Therefore, even if the platform matches a passenger in a shared trip, there is a probability for the passenger to reject such service (Wang et al., 2020). In this study, we consider such scenarios, where passengers choose among solo and shared trips and other modes (e.g. public transport), which could have significant repercussion on the platform's profitability and social welfare.

This paper presents a dynamic model of the on-demand mobility market, where we consider a single platform in the market that offers both ridesourcing (solo trips) and ridesharing services and pays wages to a fleet of drivers. Furthermore, this study explicitly considers the passengers to be cost and service quality sensitive, impatient, and have a choice to decline services offered by the platform. We use utility choice models to predict the passenger's decision. Inexorably, as a consequence for allowing passengers to reject the service offered by the platform (solo or shared rides), we consider a third category of trips, which we name 'forced solo trips'. That is when a passenger accepts the shared trip, but their counterpart rejects, then they might end up in a solo trip but still enjoys the price discount (the platform is assumed to have to honour the offer no matter what).

We propose a dynamic discount pricing strategy for the platform that is integrated in the matching algorithm which maximizes the profit with given batches of waiting passengers and idle vehicles, and strategically offers shared trips to *selected* passengers

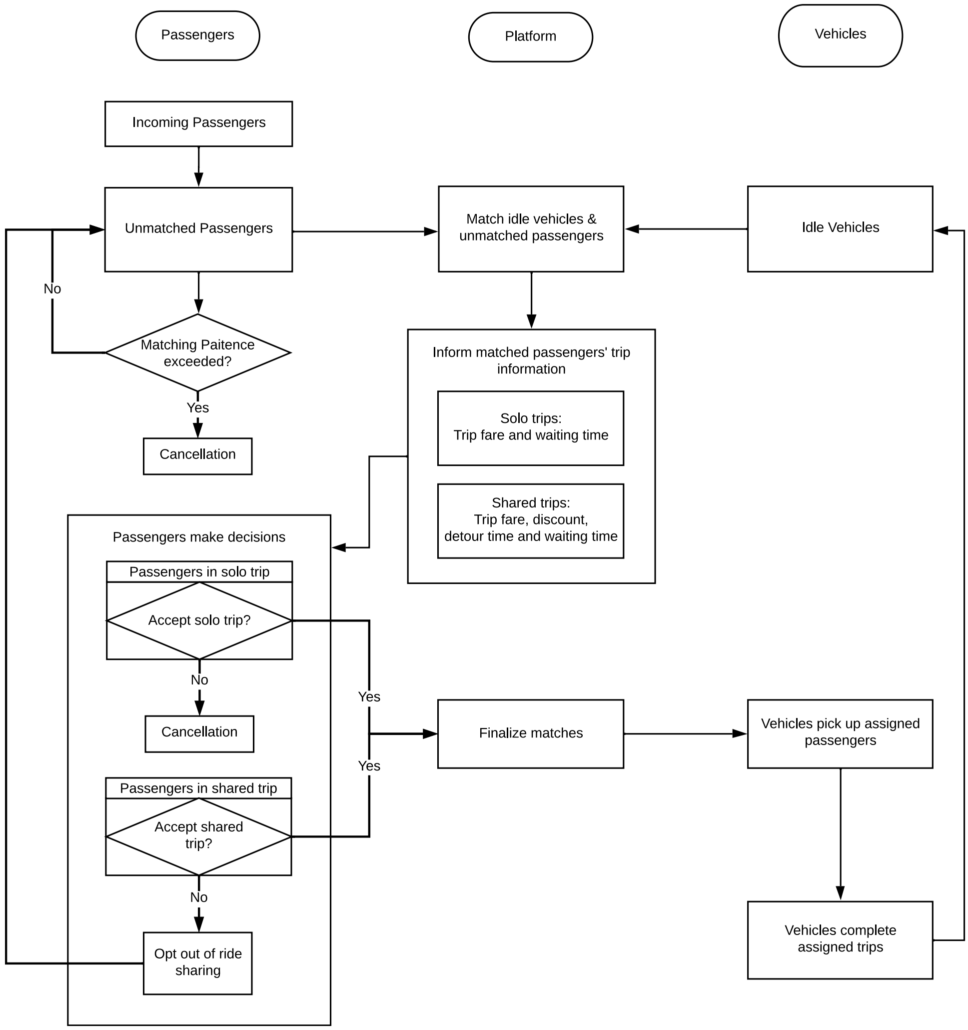


Fig. 1. Schematic diagram of interactions among market participants: passengers, vehicles, and the platform.

with varying price discount. The strategy also automatically reduces deadheadings and detour, as they incur extra wage costs paid to the drivers, which therefore are discouraged non-explicitly by the strategy. The strategy is in the form of a non-linear mixed-integer optimization problem. We then transform the problem into two sequential sub-problems that can be solved with limited computational time, while we prove the transformation is equivalent. We compare the strategy with the case where the platform offers shared trips at a constant price discount for all passengers at all times. We also analyse spatial–temporal patterns of the proposed discounting strategy.

The remainder of this paper is structured as follows. In Section 2, we present a dynamic model of the on-demand mobility market, and elaborate the interactions among all market participants. In Section 3, we formulate the matching problem and develop the dynamic discount pricing strategy. In Section 4, we present and discuss the results. Finally in Section 5, we conclude the study and draw potential future research directions.

2. Dynamic model of on-demand mobility market

The dynamic model of the on-demand mobility market considers the interactions among market participants, including individual passengers, vehicles, and the platform. A schematic diagram of their interactions is shown in Fig. 1. For each of the three groups of participants, we dedicate a section explaining how their behaviours are modelled.

2.1. The passengers

We assume passengers are impatient, cost sensitive, service quality sensitive, and heterogeneous in willingness to share. Passengers join the platform (without loss of generality in groups of one person) by requesting a ride indicating their origin and

Table 1
Trip information shown to $p_i \in \mathcal{P}$ by the platform when they are matched.

		Description	Unit
Solo trip	w_i^{s}	Waiting time	s
	f_i^{s}	Trip fare	$
Shared trip	w_i^{h}	Waiting time	s
	δ_i	Detour time	s
	f_i^{h}	Trip fare	$
	θ_i	Discount level	–

destination. Let us denote the set of all unmatched passengers $\mathcal{P} = \{p_1, \dots, p_n\}$, and an individual passenger $p_i \in \mathcal{P}$. Once the request is made, the platform will attempt to match p_i to a suitable trip. If the platform fails to match p_i in a timely manner, then their matching patience, $\bar{m}_i$, may be exceeded. Subsequently, they would cancel their request and leave the platform (i.e. travel by other modes or choose not to travel); we note it as type I cancellation. If successfully matched, the service *offered* to p_i could *either* be a solo trip or a shared-trip at the will of the platform (with the overall objective of maximizing the platform profit, see Section 3). The details of the trip would be presented to p_i by the platform, which are listed in Table 1. However, given the information, p_i may or may not be pleased with the service offered by the platform, and therefore they could choose to reject the offer. For shared trips, since there are discounts associated, we assume passengers would consider them as promotional offers and thus would not leave the platform when rejecting. Instead, those who rejected shared trips would be back into the set of unmatched passengers, and the platform would again attempt to match them but only in solo trips hereafter, as shown in Fig. 1. On the contrary, the passengers would leave the platform if they are not satisfied with the solo trip offered; we note it as type II cancellation.

It is apparent that passenger p_i may need to make two choices. (1) When offered a shared trip by the platform, they make a choice between accepting it or they would rather be in a solo trip. (2) When matched in a solo trip, they make a choice between accepting or leaving the platform for other modes of travel. We consider two binary choice models for these two scenarios. To facilitate the choice modelling, we assume p_i perceives the utilities of different modes of travel as follows:

$$\text{Solo Trip}: \ u_i^{\mathrm{s}} = \beta_i^{\mathrm{s}} - \beta_i^{\mathrm{t}} w_i^{\mathrm{s}} - \beta_i^{\mathrm{f}} f_i^{\mathrm{s}} \tag{1a}$$

$$\text{Shared Trip}: \ u_i^{\mathrm{h}} = \beta_i^{\mathrm{h}} - \beta_i^{\mathrm{t}}(w_i^{\mathrm{h}} + \delta_i) - \beta_i^{\mathrm{f}}[f_i^{\mathrm{s}}(1-\theta_i)] \tag{1b}$$

$$\text{Other Modes}: \ u_i^{\mathrm{o}} = \beta_i^{\mathrm{o}} \tag{1c}$$

where β_i^{s}, β_i^{h}, and β_i^{o} are the utility constants for the three travel modes, and β_i^{t} and β_i^{f} are the utility coefficient per unit time and cost respectively for p_i. Note here that the shared trip fare, f_i^{h}, can be substituted with $f_i^{\mathrm{s}}(1-\theta_i)$, as in Eq. (1b). The fare structures are further elaborated in Section 2.3.2.

When presented with a shared trip, if the waiting time shown exceeds the waiting patience of the passenger for a shared trip, i.e. $w_i^{\mathrm{h}} > \bar{w}_i^{\mathrm{h}}$, the passenger declines straightway. Otherwise, the passenger considers the trade-off between the shared trip and a solo trip, where the shared trip has a lower service quality with detour, potentially longer wait and discomfort but is compensated with a price discount. However, when receiving a shared trip offer, the passenger does not have the exact information regarding the solo trip they could otherwise be matched in. Therefore, to determine the utility of the solo trip, the waiting time of the solo trip, w_i^{s}, needs to be substituted with the passenger's expected waiting time, ϵ_i^{w}, and thus Eq. (1a) becomes:

$$\hat{u}_i^{\mathrm{s}} = \beta_i^{\mathrm{s}} - \beta_i^{\mathrm{t}} \epsilon_i^{\mathrm{w}} - \beta_i^{\mathrm{f}} f_i^{\mathrm{s}}. \tag{2}$$

Therefore, the probability of the passenger accepting the shared ride, $\mathrm{Pr}_i^{\mathrm{h}}$, is as follows:

$$\mathrm{Pr}_i^{\mathrm{h}} = \frac{e^{u_i^{\mathrm{h}}}}{e^{u_i^{\mathrm{h}}} + e^{\hat{u}_i^{\mathrm{s}}}}. \tag{3}$$

Similarly, when presented with a solo trip, the passenger declines if the waiting patience for a solo trip is exceeded, i.e. $w_i^{\mathrm{s}} > \bar{w}_i^{\mathrm{s}}$. Otherwise, the passenger makes a binary choice between accepting the solo trip or other modes of travel, where the probability of accepting the solo ride $\mathrm{Pr}_i^{\mathrm{s}}$, is as follows:

$$\mathrm{Pr}_i^{\mathrm{s}} = \frac{e^{u_i^{\mathrm{s}}}}{e^{u_i^{\mathrm{s}}} + e^{u_i^{\mathrm{o}}}}. \tag{4}$$

After accepting a trip, no more action is executed by passenger p_i. They wait to be picked up by their designated vehicle, their status change from unmatched to matched, are removed from the set $\mathcal{P}$, and will leave the platform once reaching their destination.

2.2. The vehicles

The drivers/vehicles are the supply side of the on-demand mobility market. Their characteristics such as entering to and exiting from the market, ability to decline rides, incentivization for repositioning are considered by the literature. Since this study is focused on passenger modelling and demand manipulation, we assume that vehicles will always accept the designated trips assigned by the platform. The vehicles travel via shortest path (e.g. Dijkstra's algorithm) to specific locations under the direction of the platform.

The effects of congestion is not considered in this study; we assume that every link in the road network has a specific speed and vehicles travel at those speeds. We also assume the vehicles have a maximum capacity of two passengers, and they will be stationary when all passengers are dropped off. Therefore each vehicle could be in one of the four possible states:

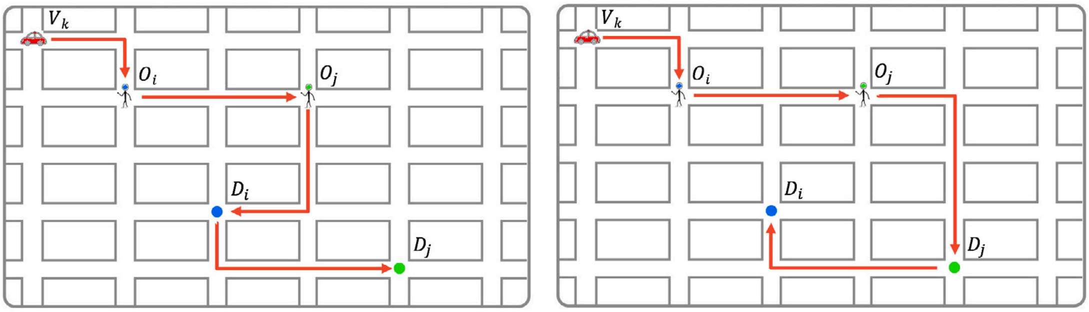
- Idle: unoccupied and stationary, $\mathcal{V} = \{v_1, \dots, v_v\}$;
- Dispatched: unoccupied but on the way to pick up passengers;
- Partially Occupied: occupied by one passenger;
- Fully Occupied: occupied by two passengers.

2.3. The platform

The platform has the information of unmatched passengers' requests (origin and destination) and idle vehicles' positions, and will determine optimal matching between the two batches every Δ seconds (note that partially occupied vehicles are not considered for matching in this study). This section will first discuss the composition of a shared trip and outline the pricing schemes of the platform. Then the matching procedure will be presented, while detailed matching algorithm will be elaborated in Section 3.

2.3.1. Shared trips

Let us consider any two unmatched passengers, $p_i, p_j \in \mathcal{P}$, and any idle vehicle, $v_k \in \mathcal{V}$. There are four possible arrangements of a shared trip. We denote the position of v_k as V_k, and the origins and destinations of the two passengers, as $[O_i, O_j]$ and $[D_i, D_j]$, respectively. Consider p_i is the first passenger, then the two possible sequences are First In First Out (FIFO) and Last In First Out (LIFO), which are shown in Fig. 2 respectively. Similarly, there are another two sequences if p_j is the first passenger.



(a) FIFO: $V_k \to O_i \to O_j \to D_i \to D_j$ (b) LIFO: $V_k \to O_i \to O_j \to D_j \to D_i$

Fig. 2. Shared trip pick-up & drop-off sequences.

Given the sequence, we can determine the waiting time and detour time for the passengers explicitly. We define the waiting time as the time between matching notification and pick-up. We also define the detour time for the passengers as the difference between actual on-board time and non-shared original on-board time. Furthermore, we denote the shortest path travel time from O to D as $|OD|$. Accordingly, the waiting time and detour time for the two passengers in the four possible sequences are summarized in Table 2.

Table 2
Passenger waiting time and detour time for all sequences of shared trips.

		FIFO	LIFO
First passenger	p_i	$w_i^h = \lvert V_k O_i \rvert$ $w_j^h = \lvert V_k O_i \rvert + \lvert O_i O_j \rvert$ $\delta_i = \lvert O_i O_j \rvert + \lvert O_j D_i \rvert - \lvert O_i D_i \rvert$ $\delta_j = \lvert O_j D_i \rvert + \lvert D_i D_j \rvert - \lvert O_j D_j \rvert$	$w_i^h = \lvert V_k O_i \rvert$ $w_j^h = \lvert V_k O_i \rvert + \lvert O_i O_j \rvert$ $\delta_i = \lvert O_i O_j \rvert + \lvert O_j D_j \rvert + \lvert D_j D_i \rvert - \lvert O_i D_i \rvert$ $\delta_j = 0$
	p_j	$w_i^h = \lvert V_k O_j \rvert + \lvert O_j O_i \rvert$ $w_j^h = \lvert V_k O_j \rvert$ $\delta_i = \lvert O_i D_j \rvert + \lvert D_j D_i \rvert - \lvert O_i D_i \rvert$ $\delta_j = \lvert O_j O_i \rvert + \lvert O_i D_j \rvert - \lvert O_j D_j \rvert$	$w_i^h = \lvert V_k O_j \rvert + \lvert O_j O_i \rvert$ $w_j^h = \lvert V_k O_j \rvert$ $\delta_i = 0$ $\delta_j = \lvert O_j O_i \rvert + \lvert O_i D_i \rvert + \lvert D_i D_j \rvert - \lvert O_j D_j \rvert$

2.3.2. Pricing and cost

For a solo trip, we assume that the fare structure consists of a fixed cost and a variable cost that depends on trip distance/duration as:

$$f_i^s = \alpha_1 + \alpha_2 |O_i D_i|. \tag{5}$$

For a shared trip, we assume that a passenger will receive a percentage based discount off the original solo trip fare:

$$f_i^h = f_i^s (1 - \theta_i). \tag{6}$$

Since we assume that drivers always accept the designated trips by the platform, then it would be unreasonable for a driver to have an excessive deadheading. Therefore we assume that payments to the driver k, c^{sq}_{ijk} and c_{ik} for shared trips of given sequence and solo trips respectively, are based on the travel time from their idle position to last drop-off position. E.g. for the cases in Figs. 2(a), 2(b), and a solo trip, their costs are shown in Eq. (7) respectively:

$$c^{\mathrm{sq}}_{ijk} = \begin{cases} \alpha_3(|V_kO_i| + |O_iO_j| + |O_jD_i| + |D_iD_j|) & \text{if sq = FIFO} \\ \alpha_3(|V_kO_i| + |O_iO_j| + |O_jD_j| + |D_jD_i|) & \text{if sq = LIFO} \end{cases} \tag{7a}$$

$$c_{ik} = \alpha_3(|V_kO_i| + |O_iD_i|). \tag{7b}$$

It can be observed that the term $|V_kO_i|$, which represents the time Vehicle k travels without any passengers, appears in all three equations in Eq. (7). Therefore, under such wage structure, for a profit seeking platform, deadheading will be inherently reduced, which inadvertently benefits the passengers and drivers.

We want to point out that, though currently most of the ride-sourcing drivers are not paid for the deadheading in real word applications, we still decided to use this wage structure in the study with much consideration. Such wage structure is an ingenious way to (partially and indirectly) kerb the wild goose chase (WGC) problem, where vehicles might be dispatched to passengers considerably far away when demand and/or supply is low, which reduces the efficiency of the system for all stakeholders. To address the WGC problem, one way is to introduce an artificial search radius (in distance or time) where the platform will not consider matching a passenger outside of a particular vehicle's search radius (Alonso-Mora et al., 2017). However, under the proposed wage structure, picking up passengers far away would incur extra cost to the platform, therefore, it implicitly addresses the WGC problem and could lead to a more efficient operation.

It follows that, the profit made by the platform on each trip, π^{sq}_{ijk} and π_{ik} for shared and solo trips respectively, is the difference between the trip fare and payment to the drivers:

$$\pi^{\mathrm{sq}}_{ijk} = f^{\mathrm{h}}_i + f^{\mathrm{h}}_j - c^{\mathrm{sq}}_{ijk} \tag{8a}$$

$$\pi_{ik} = f^{\mathrm{s}}_i - c_{ik}. \tag{8b}$$

We emphasize here that θ_i in Eq. (6) is a decision variable. The higher θ_i, the higher the probability for passenger p_i to accept the shared trip (Eq. (3)), but inexorably, the lower the profit made by the platform on the shared trip (Eq. (8a)), ceteris paribus. Therefore offering the 'optimal' discount is a crucial decision making process for the platform, which is discussed in detail in Section 3.

2.3.3. Matching procedure

For a given set of unmatched passengers $\mathcal{P} = \{p_1, \dots, p_n\}$, $\mathcal{P}$ is comprised of two disjoint subsets, $\mathcal{P}_1 = \{p_1, \dots, p_m\}$ and $\mathcal{P}_2 = \{p_{m+1}, \dots, p_n\}$, where $\mathcal{P}_1$ consists of all unmatched passengers who have not been matched previously and thus can be matched in either shared or solo trips. Whereas $\mathcal{P}_2$ consists of all unmatched passengers who have previously rejected a shared trip offer whom shall only be considered in solo trips by the platform. The platform matches $\mathcal{P}_1$ and $\mathcal{P}_2$ to the set of idle vehicles, $\mathcal{V} = \{v_1, \dots, v_v\}$. The matching algorithm produces the optimal matching (in the sense of maximizing the platform profit) which consists of a number of shared and solo trips as well as the optimal price discount for each passenger in shared trips. In the optimal matching, any passenger can only exist in at most one of the trips, and any vehicle can also only exist in at most one of the trips. For all passengers who are matched, they will receive the corresponding trip information, which will lead to the choice modelling already discussed in Section 2.1.

For all solo passengers who accepted, and passenger pairs that both accepted their rides, the platform will dispatch the designated vehicles to pick up and complete their trips. However, a consequence of allowing passengers to reject is that there will be those who accept the shared trips but their counterparts reject, which will force them to be in solo trips. We assume that under such scenario, the platform will dispatch the original vehicle that was matched to the pair to pick up only the one passenger who accepted, while still honouring the price discount offered to him/her, even if the platform makes a loss on the trip.

We considered a rematch algorithm where the platform may be able to mitigate a portion of such losses by considering a rematch between the smaller set of passengers who accepted their rides, $\hat{\mathcal{P}}$, and the idle vehicles, $\mathcal{V}$. We found the performance of this algorithm is not significant, and it can produce an adverse effect in certain scenarios. Therefore, we did not include this algorithm in the main body of this study; nonetheless the relevant results are shown in Appendix C.

3. Dynamic discount method

3.1. Problem description

The platform establishes matching between a set of unmatched passengers, $\mathcal{P} = \{p_1, \dots, p_n\}$, and a set of idle vehicles, $\mathcal{V} = \{v_1, \dots, v_v\}$ every Δ seconds. The dynamic discount pricing strategy is designed to determine (i) the profit maximizing matching between the two sets that consists of solo and shared trips, and simultaneously (ii) the profit maximizing fare discount for every passenger in shared trips. We assume that the platform:

- distinguishes $\mathcal{P}_1$ and $\mathcal{P}_2$, where $\mathcal{P}_1$ consists of all unmatched passengers who have not been matched previously, whom can be matched in either shared or solo trips, and $\mathcal{P}_2$ consists of all unmatched passengers who have previously rejected a shared trip offer whom shall only be considered in solo trips by the platform,

- is aware of the origins and destinations of all passengers,
- is aware of the positions of all idle vehicles,
- is capable of determining the shortest path time between any two positions in the network,
- is aware of the population means of β_i^{s}, β_i^{h}, β_i^{o}, β_i^{t}, β_i^{f}, ϵ_i^{w},
- does not alter the fare structure of solo trips as well as salary paid to drivers, i.e. $\alpha_1, \alpha_2, \alpha_3$ are fixed,
- offers discount off the original solo trip fare to passengers in shared trips.

Section 3.2 proposes a *modified* maximum weighted bipartite matching to solve the problem. In Section 3.3, we transform the *modified* maximum weighted bipartite matching problem into two sequential sub-problems that can be solved with limited computational time. Then in Section 3.4, we prove the transformation is equivalent (i.e. does not marginalize the optimality).

3.2. Joint matching and dynamic discount optimization formulation

We establish the profit maximizing matching and discounts by solving a *modified* maximum weighted bipartite matching problem. Consider the distinct two sets of unmatched passengers, $\mathcal{P}_1 = \{p_1, \dots, p_m\}$ and $\mathcal{P}_2 = \{p_{m+1}, \dots, p_n\}$. We first construct a set of all possible ordered passenger pairs, $\mathcal{Q}$, using $\mathcal{P}_1$.

$$\mathcal{Q} = \{p_{1,2}^{\mathrm{FIFO}}, p_{1,2}^{\mathrm{LIFO}}, \dots, p_{1,m}^{\mathrm{FIFO}}, p_{1,m}^{\mathrm{LIFO}}, p_{2,1}^{\mathrm{FIFO}}, p_{2,1}^{\mathrm{LIFO}}, p_{2,3}^{\mathrm{FIFO}}, p_{2,3}^{\mathrm{LIFO}}, \dots, p_{2,m}^{\mathrm{FIFO}}, p_{2,m}^{\mathrm{LIFO}}, \dots, p_{m,1}^{\mathrm{FIFO}}, p_{m,1}^{\mathrm{LIFO}}, \dots, p_{m,m-1}^{\mathrm{FIFO}}, p_{m,m-1}^{\mathrm{LIFO}}\}$$

For consistency, we denote the set of all individual passengers, $\bar{\mathcal{P}} = \left\{p_{1,1}^-, \dots, p_{n,n}^-\right\}$. We let $\mathcal{R} = \bar{\mathcal{P}} \cup \mathcal{Q}$, then $\mathcal{R}$ is the set of all potential passenger pairs and solo passengers. Let $\mathcal{E}$ be the set of edges connecting each element of $\mathcal{R}$ and $\mathcal{V}$, where an edge $(p_{i,j}^{\mathrm{sq}}, v_k) \in \mathcal{E}$ connects $p_{i,j}^{\mathrm{sq}} \in \mathcal{R}$ and $v_k \in \mathcal{V}$. The *modified* maximum weighted bipartite matching can then be formulated as follows:

$$\max_{x_{ijk}^{\mathrm{sq}}, \theta_{i,ijk}^{\mathrm{sq}}, \theta_{j,ijk}^{\mathrm{sq}}} \sum_{(p_{i,j}^{\mathrm{sq}}, v_k) \in \mathcal{E}} g_{ijk}^{\mathrm{sq}} x_{ijk}^{\mathrm{sq}} \tag{9a}$$

$$\text{where}: \quad g_{ijk}^{\mathrm{sq}} = \begin{cases} E[\pi_{ik}] & \text{if } i = j \text{ or sq} = - \text{ (solo trip)} \\ E[\pi_{ijk}^{\mathrm{sq}}] = F(\theta_{i,ijk}^{\mathrm{sq}}, \theta_{j,ijk}^{\mathrm{sq}}) & \text{if } i \neq j \text{ (shared trip)} \end{cases} \tag{9b}$$

s.t.

$$\sum_{\mathrm{sq} \in \{\mathrm{FIFO,LIFO}\}} \sum_{j=1}^{n} \sum_{k=1}^{v} x_{\hat{i}jk}^{\mathrm{sq}} + \sum_{\mathrm{sq} \in \{\mathrm{FIFO,LIFO}\}} \sum_{i=1}^{n} \sum_{k=1}^{v} x_{i\hat{j}k}^{\mathrm{sq}} + \sum_{k=1}^{v} x_{\hat{i}\hat{j}k}^{-} \leq 1 \quad \forall\, \hat{i} = \hat{j} \in \mathcal{P} \tag{9c}$$

$$\sum_{\mathrm{sq} \in \{\mathrm{FIFO,LIFO},-\}} \sum_{i=1}^{n} \sum_{j=1}^{n} x_{ijk}^{\mathrm{sq}} \leq 1 \quad \forall k\,;\, v_k \in \mathcal{V} \tag{9d}$$

$$x_{ijk}^{\mathrm{sq}} \in \{0, 1\} \tag{9e}$$

$$\theta_{i,ijk}^{\mathrm{sq}}, \theta_{j,ijk}^{\mathrm{sq}} \in [0, 1] \tag{9f}$$

In Eq. (9a), x_{ijk}^{sq} is a binary decision variable on the edge $(p_{i,j}^{\mathrm{sq}}, v_k) \in \mathcal{E}$, with $x_{ijk}^{\mathrm{sq}} = 1$ indicating that the solo passenger p_i (if $i = j$), or the passenger pair p_i and p_j of given sequence sq (if $i \neq j$ or sq $\neq -$) are matched to vehicle v_k. g_{ijk}^{sq} is the weight of the edge, as shown in Eq. (9b); we let it be the expected profit made by the platform. Thus, for a shared trip, g_{ijk}^{sq} is a function of $\theta_{i,ijk}^{\mathrm{sq}}$ and $\theta_{j,ijk}^{\mathrm{sq}}$, which are the discount rates offered to passengers i and j respectively for the given sequence when they are matched to vehicle k. Contrary to most maximum weighted bipartite matching problems, where the weights of the edges are known, we formulate our problem such that each edge weight of shared arrangements is dependent on a set of decision variables, $\theta_{i,ijk}^{\mathrm{sq}}$ and $\theta_{j,ijk}^{\mathrm{sq}}$. Note that the discount level is between 0 and 1 (see Eq. (9f)), as we assume the platform cannot charge more for the shared trip than the original price, and it cannot give money to customers in shared trips. We define $E[\pi_{ik}]$ and $E[\pi_{ijk}^{\mathrm{sq}}]$ explicitly in Appendix A. Eqs. (9c) and (9d) are constraints that ensure each idle vehicle and unmatched passenger are matched at most once.

By solving this *modified* maximum weighted bipartite matching problem, the profit maximizing matching and the profit maximizing individualized price discount can be determined simultaneously. However, solving this non-linear mixed-integer optimization problem by brute force is inevitably computationally expensive. Therefore, in Section 3.3 we transform the problem into two sequential sub-problems that can be solved with limited computational time. Then in Section 3.4 we prove the transformation is equivalent.

3.3. Transformation of optimization formulation

Problem 1. First we determine the optimal fare discount for the two passengers in all possible shared trip combinations (including different vehicles), which maximizes the expected profit of such combination, i.e.

$$(\theta_{i,ijk}^{\mathrm{sq}}{}^*, \theta_{j,ijk}^{\mathrm{sq}}{}^*) = \arg\max_{\theta_{i,ijk}^{\mathrm{sq}}, \theta_{j,ijk}^{\mathrm{sq}}} E[\pi_{ijk}^{\mathrm{sq}}] \quad \forall i \neq j \in \mathcal{P} \;\; \forall k\,;\, v_k \in \mathcal{V} \tag{10a}$$

$$\text{s.t. } \theta_{i,ijk}^{\mathrm{sq}}, \theta_{j,ijk}^{\mathrm{sq}} \in [0, 1] \tag{10b}$$

The optimization problem in Eq. (10) finds the maximum of the bi-variate function, $F(\theta^{sq}_{i,ijk}, \theta^{sq}_{j,ijk})$, within the domain of $\theta^{sq}_{i,ijk}, \theta^{sq}_{j,ijk} \in [0,1]$, which needs to be solved for all shared trip combinations. Due to our definition of F, which is the expected profit of the arrangement, as shown in Eq. (13), the explicit solution to Eq. (10) is unattainable. Therefore, the problem needs to be solved non-explicitly for all shared trip combinations, which can be computationally expensive. For efficiency, we solve each optimization problem by adopting a grid search approach, where x equally spaced points between 0 and 1 are taken for each of $\theta^{sq}_{i,ijk}$ and $\theta^{sq}_{j,ijk}$, creating x^2 possible discount combinations. The solution is then obtained by the combination that yields the highest expected profit. In this study, we choose x to be 21, that is the discount intervals are at 0.05. We believe this interval is adequate, as calculations can be done efficiently (2.6 GHz Intel Core i7: solving for one arrangement/edge takes 0.00001 s; solving for one full instance with 560 empty vehicles and 400 ordered passenger pairs, i.e. 2,24,000 arrangements, takes 2.53 s), while the interval is sufficiently small, i.e. for a $10 trip, the possible discounts are 50 cents apart. Note that the size of x can be altered to improve accuracy but at the cost of efficiency, and vice versa; or that the objective function, F, can be chosen differently such that an explicit solution can be obtained.

Problem 2. After solving Problem 1, we use its results (i.e. θ^* and $E^*[\pi^{sq}_{ijk}]$) to determine the edge weights. Therefore, Problem 2 can be formulated as a maximum weighted bipartite matching problem with known edge weights:

$$\max_{x^{sq}_{ijk}} \sum_{(p^{sq}_{i,j}, v_k)\in\mathcal{E}} g^{sq\,*}_{ijk}\, x^{sq}_{ijk} \tag{11a}$$

$$\text{where}: \quad g^{sq\,*}_{ijk} = \begin{cases} E[\pi_{ik}] & \text{if } i=j \text{ or sq} = - \text{ (solo trip)} \\ E^*[\pi^{sq}_{ijk}] = F(\theta^{sq\,*}_{i,ijk}, \theta^{sq\,*}_{j,ijk}) & \text{if } i\neq j \text{ (shared trip)} \end{cases} \tag{11b}$$

s.t.

$$\sum_{sq\in\{\text{FIFO,LIFO}\}} \sum_{j=1}^{n}\sum_{k=1}^{v} x^{sq}_{\hat{i}jk} + \sum_{sq\in\{\text{FIFO,LIFO}\}} \sum_{i=1}^{n}\sum_{k=1}^{v} x^{sq}_{i\hat{j}k} + \sum_{k=1}^{v} x^{-}_{\hat{i}\hat{j}k} \le 1 \quad \forall\, \hat{i}=\hat{j}\in\mathcal{P} \tag{11c}$$

$$\sum_{sq\in\{\text{FIFO,LIFO},-\}} \sum_{i=1}^{n}\sum_{j=1}^{n} x^{sq}_{ijk} \le 1 \quad \forall k;\ v_k\in\mathcal{V} \tag{11d}$$

$$x^{sq}_{ijk} \in \{0,1\} \tag{11e}$$

Problem 2 is an integer linear programming problem which can be solved by state-of-the-art algorithms efficiently (Lenstra, 1983). Solving Problem 2, the decision variables, $x^{sq}_{ijk} = 1$ indicates that arrangement $p^{sq}_{i,j}$ and v_k is in the profit maximization matching. Furthermore, if $i \neq j$, then the arrangement is a shared trip, and the profit maximizing discounts for the two passengers are $\theta^{sq\,*}_{i,ijk}$ and $\theta^{sq\,*}_{j,ijk}$ respectively.

The question that remains is that whether solving for optimal discount and optimal matching sequentially (transformed method), as shown in this section, is equivalent to solving them simultaneously as formulated in Eq. (9). We shall prove their equivalence in the following section.

3.4. Proof of equivalency

If we assume that the maximization yielded by solving Problems 1 and 2 sequentially is not equivalent to Eq. (9), then there must exist at least one passenger pair (or solo passenger) that is matched when solving Eq. (9), whom will not be matched or receive a different level of discount when solved by the transformed methods.

Let us look at an arbitrary passenger pair of given sequence, say $(p^{sq}_{i,j}, v_k) \in \mathcal{E}$, whose decision variable, x^{sq}_{ijk}, equals 1 in the solution of Eq. (9), i.e. this arrangement is in the profit maximization matching, and the corresponding discount levels in the solution are $\theta^{sq}_{i,ijk}$ and $\theta^{sq}_{j,ijk}$ respectively. Then we look at this particular edge in Eq. (10), the results $\theta^{sq\,*}_{i,ijk}$ and $\theta^{sq\,*}_{j,ijk}$ must be equivalent to $\theta^{sq}_{i,ijk}$ and $\theta^{sq}_{j,ijk}$. This is because we cannot have $\theta^{sq}_{i,ijk}$ and $\theta^{sq}_{j,ijk}$ that yields a higher $E[\pi^{sq}_{ijk}]$ than $\theta^{sq\,*}_{i,ijk}$ and $\theta^{sq\,*}_{j,ijk}$, as by definition $\theta^{sq\,*}_{i,ijk}$ and $\theta^{sq\,*}_{j,ijk}$ yields the maximum $E[\pi^{sq}_{ijk}]$. We also cannot have $\theta^{sq}_{i,ijk}$ and $\theta^{sq}_{j,ijk}$ that yields a lower $E[\pi^{sq}_{ijk}]$ than $\theta^{sq\,*}_{i,ijk}$ and $\theta^{sq\,*}_{j,ijk}$, as we can unilaterally change $\theta^{sq}_{i,ijk}$ and $\theta^{sq}_{j,ijk}$ to $\theta^{sq\,*}_{i,ijk}$ and $\theta^{sq\,*}_{j,ijk}$ in the solution of Eq. (9) and yield a higher maximum, which again by definition cannot happen.

The same logic can be applied to all passenger pairs matched when solving Eq. (9); their respective θ^*s when solving Eq. (10), will not be different to θs in the solution of (9). Now if we consider the subset of all passenger pairs and solo passengers that are matched in Eq. (9) and the subset of vehicles they are matched to. We solve Eq. (11) based on these two subsets, and the edge weights which are just proved to be equivalent to that in the solution of Eq. (9). Then the results, i.e. which pair/solo passenger is matched to which vehicle, must also be equivalent to the results obtained in Eq. (9). Since if a lower maximum is yielded in Eq. (11) than in Eq. (9), then the definition of Eq. (11) is violated and vice versa.

Finally, consider solving Eq. (11) with the full set of passenger pairs, solo passengers and vehicles. The resulting maximum must be greater or equal to the maximum when the problem is solved with the subsets. However, if a greater maximum is achieved, then this maximum value will also be higher than that obtained in Eq. (9). It would imply that the transformation yields better optimization than the original problem, which violates the definition of Eq. (9). Hence the solution of Eq. (11) is equivalent to all x^{sq}_{ijk} in the solution of Eq. (9).

Therefore, we have shown that for all passenger pairs (or solo passengers) matched when solving Eq. (9), they will also be matched and receive the same level of discount when solved by using the transformed method. Hence, we show that solving Problems 1 and 2 sequentially is equivalent to Eq. (9).

Note that we use a grid search approach to solve Eq. (10), therefore the results obtained is not the exact solution but an approximation. The transformed method is only equivalent if we allow the same level of discretization for the discounts in Eq. (9). We also point out that the problem transformation is only possible due to the edge weights being independent of each other. In other words, what the platform offers to one passenger pair has no effect on the discounts offered to other passenger pairs. This assumption may not always hold, if for example, the platform has a quota on the amount of discount that can be offered to the passengers.

4. Numerical experiments

We test the proposed dynamic discount method in a simulator that considers the road network in Manhattan New York as a directed graph. We use real passenger demand data from 7–11 am (4 h) on five different weekdays (3–6/Feb/2015 and 9/Feb/2015). In Section 4.1, we detail the experiment setup. In Section 4.2, we present a benchmark strategy where the platform offers a constant discount level to all passengers matched in shared-trips and evaluate the effectiveness of the dynamic discount method by comparing its performance to the benchmark. In Section 4.3, we look more closely at the discount levels offered by the dynamic discount strategy. Note that in Sections 4.2 and 4.3, we display the results obtained with one set of demand data (3/Feb/2015), since the results for all five days exhibit similar trends for both the benchmark and the dynamic discount strategy.

4.1. Experiment setup

At the beginning of each simulation, vehicles are incrementally added at random locations on the road network. We consider a fleet of 4200 vehicles with an addition rate of 3 vehicles per second. It takes 1400 s for all vehicles to be added. We allow another 1000 s afterwards as a warm-up period. Therefore, any measurements are taken after a total of 40 min.

We have introduced a number of parameters regarding passenger choice in Section 2.1. The parameters for each individual passenger are drawn from bounded normal distributions. The population means (μ) (utility coefficients are based on the studies by Habib (2019) and Lavieri and Bhat (2019)), standard deviations (σ), and lower and upper bounds (a and b) are shown in Table 3. Note that the strategy only uses the population mean values in the choice model (e.g. β^t, β^f, etc.), while each individual in the experiments has a unique choice and patience characteristics (e.g. β_i^{t}, β_i^{f}, etc.).

For the platform, we assume that it adopts a matching interval of 10 s, i.e. $\Delta = 10s$. Furthermore, the default parameter values for the pricing and cost structure (Section 2.3.2) are shown in Table 4. Under such setup, series of experiments were conducted to produce the benchmark results (i.e. with fixed discounts) and to evaluate the effectiveness of the dynamic discount strategy.

Table 3
Passenger choice parameters: population mean, standard deviation, & lower and upper bounds.

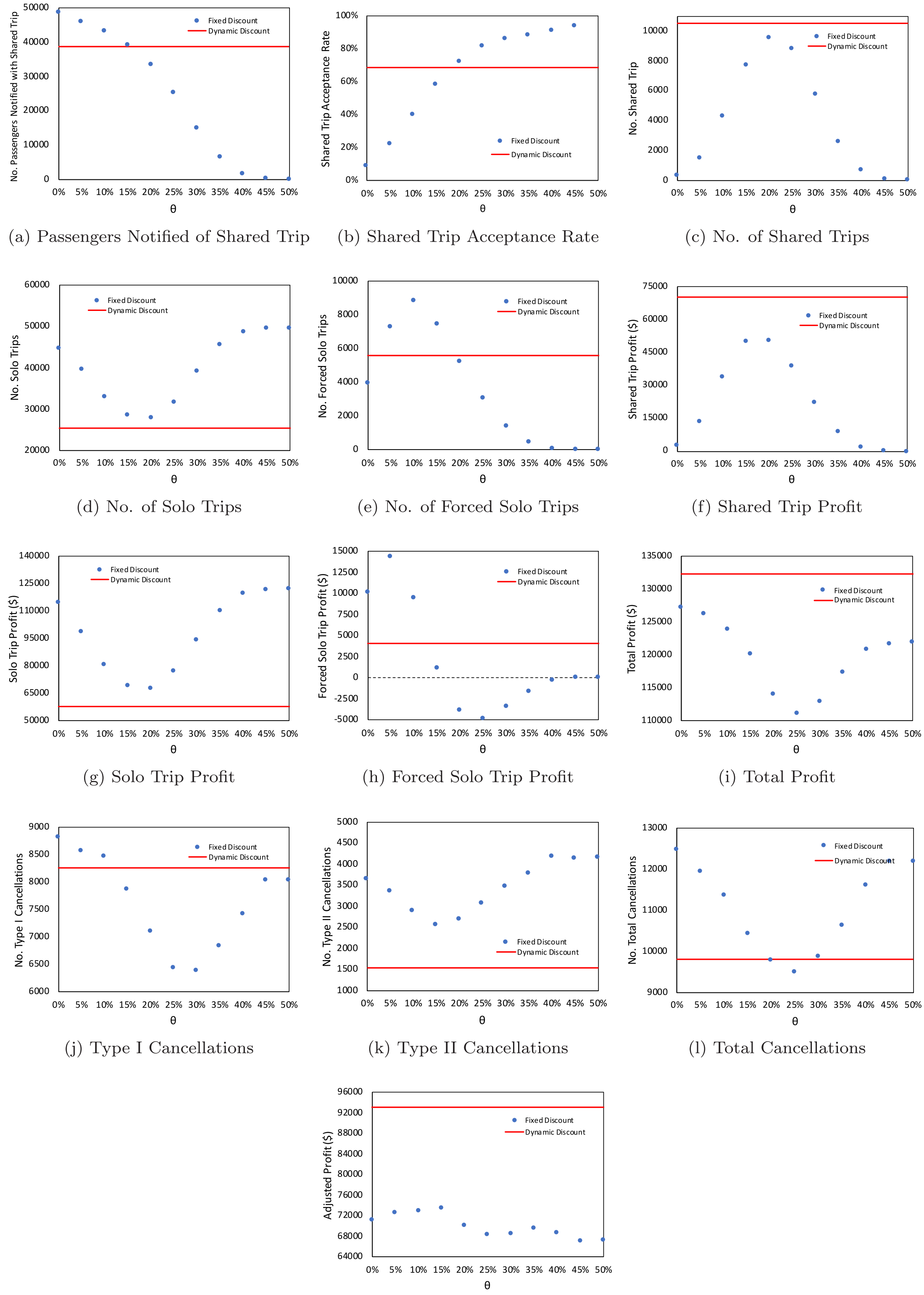
Parameter	Description	Unit	μ	σ	a	b
$\bar{m}_i$	Matching patience of p_i	s	60	10	40	80
$\bar{\omega}_i^{\mathrm{s}}$	Waiting patience for solo trip of p_i	s	480	30	420	540
$\bar{\omega}_i^{\mathrm{h}}$	Waiting patience for shared trip of p_i	s	600	60	480	720
ϵ_i^{w}	Expected waiting time of p_i	s	240	30	180	300
β_i^{t}	Utility coefficient for total travel time of p_i	1/min	0.6	0.05	0.5	0.7
β_i^{f}	Utility coefficient for trip fare of p_i	1/$	3.2	0.2	2.8	3.6
β_i^{s}	Utility constant for solo trip of p_i	–	0	0	–	–
β_i^{h}	Utility constant for shared trip of p_i	–	−0.8	0.1	−1	−0.6
β_i^{o}	Utility constant for other travel mode of p_i	–	−60	10	−80	−40

Table 4
Parameter values for pricing and cost.

Parameter	Description	Unit	Value
α_1	Fare base rate	$	2.55
α_2	Trip duration variable rate	$/min	0.6
α_3	Per unit time driver salary	$/min	0.48

4.2. Constant discount vs dynamic discount

We consider benchmark scenarios where the platform offers a constant discount level to all passengers matched in shared-trips. That is, for the given set of unmatched passengers and idle vehicles, instead of solving Eq. (10) and Eq. (11), we solve a version of Eq. (11) only. We assume ${\theta_{i,ijk}^{\mathrm{sq}}}^* = {\theta_{j,ijk}^{\mathrm{sq}}}^* = \theta$, where θ remains constant through each simulation, and instead of using expected profit as edge weight, we use the actual profit in Eq. (11b). We conduct experiments for varying θ between the range from 0 to 0.5 (i.e. 0 to 50% discount for sharing). It may be noted that under the benchmark setup, it is illogical for the platform to offer passengers in shared trips a discount of 50% or above, since it is always more or equally profitable for the platform to just serve a solo trip at full price to one of the passenger in a shared trip. Therefore, for all constant discount levels greater than and equal to



(a) Passengers Notified of Shared Trip (b) Shared Trip Acceptance Rate (c) No. of Shared Trips

(d) No. of Solo Trips (e) No. of Forced Solo Trips (f) Shared Trip Profit

(g) Solo Trip Profit (h) Forced Solo Trip Profit (i) Total Profit

(j) Type I Cancellations (k) Type II Cancellations (l) Total Cancellations

(m) Adjusted Profit

Fig. 3. Results of constant discount scenarios (Blue) and the results of the dynamic discount pricing strategy (Red). The x axis corresponds to the varying benchmark levels of the constant discount, θ, which is not associated with the dynamic discount pricing strategy. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

50%, i.e. $\theta \geq 0.5$, they are exactly the same scenario where the platform offers ridesourcing services only. We compare the results of the benchmarks with the dynamic discount pricing strategy in Fig. 3.

Fig. 3(a) shows the total number of passengers that were notified with shared trips in each simulation at varying levels of θ. It can be observed that at low levels of discount, as the platform is more likely to find shared trips to be more profitable than solo trips, more passengers will be matched in shared trips. As the discount level increases, shared trips become less profitable, and thus fewer shared trips will be organized by the platform. When the discount level is at 50%, for the reasons given above, none of the passengers are matched in shared trips by the platform. Correspondingly in Fig. 3(b), the average rate at which passengers accept the offered shared trips are shown (no data point at 50% discount since there is 0 passengers offered a shared trip). It is intuitive that when the discount is low, though more passengers are matched in shared trips, only a small fraction will accept the shared trips and vice versa. Consequently, the total number of shared trips are shown in Fig. 3(c), where we see a distinct peak at 20% discount level; whereas much fewer shared trips are conducted at the two extreme ends of the discount range. Comparing to the dynamic discount pricing strategy, the number of passengers notified of shared trips is roughly equivalent to the benchmark scenario of 15% discount, however the level of acceptance is higher, which resulted in a 36.6% increase in the number of shared trips comparing to the 15% discount benchmark.

The number of solo trips are shown in Fig. 3(d). The shape of the plot for the benchmarks is roughly inverse of that in Fig. 3(c), as when more passenger are in shared trips, less solo trips are formed to service the remaining passengers. Forced solo trips occur when a passenger accepts the shared trip, but his/her counterpart rejects. Counter-intuitively, as shown in Fig. 3(e), the most number of forced solo trips for the benchmarks does not occur at the lowest discount level, when there are the most passengers notified with shared trip as well as the highest shared trip rejection rate. This is because when the rejection rate is at such high level, there is consequently a high likelihood of the two passengers in a planned shared trip both rejects, and thus neither would end up in a forced solo trip. Under the dynamic discount pricing strategy, fewer solo trips are conducted comparing to any benchmark since the strategy successfully promoted ridesharing. In addition, more passengers are being serviced in shared trips, while the number of forced solo trips is moderate due to a relatively high shared trip acceptance rate, which is indicative that the strategy takes into consideration the passengers' choices.

Figs. 3(f)–3(h) show the profits made by the platform for the three trip types, shared, solo, and forced solo trip respectively. The profits are highly correlated with the number of successful trips conducted of each type, as can be observed in conjunction with Figs. 3(c)–3(e) . However, the profit of forced solo trips correlates to, but does not follow exactly the trend of the number of forced solo trips. Several factors attribute to this observation. Firstly, the profitability of individual forced solo trips is negatively correlated with discount, and the variation in profitability is more drastic with changes in discount comparing to the other two trip types. Therefore the maximum of forced solo trip profits occurs at a lower discount level where the trips are more profitable comparing to when the maximum trips are conducted. Furthermore, it is possible for platform to make a loss on forced solo trips, as we assume that the platform will honour the discount offer to the remaining passenger. Therefore, as the discount increases, the platform will begin making a loss on each forced solo trip while the number of forced solo trips decreases, which results in a local minimum in the profit at 25% discount.

Combining the profit for the three types of trips, we have the total profit made by the platform in Fig. 3(i). We observe that for constant discounts, the highest profit is yielded when the platform is offering no discount at all. It is also apparent that the dynamic discount pricing strategy outperforms the benchmarks, by 4.0% comparing to the 0% discount benchmark level This increase in profit is attributed by the increase in shared trips, as shown in 3(c) and 3(f), which also has societal benefits.

However, profit alone may not paint a complete picture. The number of cancellations is another indicator that a platform should pay attention to. If a customer cancels, they may defect to other platforms and thus leading to a long-term loss. Type I cancellations, that is when a passenger is yet to receive a quote from the platform, are shown in Fig. 3(j). Type II cancellations, that is when a passenger is not satisfied with the service offered by the platform, are shown in Fig. 3(k). We distinguish the two types of cancellations as it may occur to the platform that each type has distinct implications, and either may be addressed by the platform differently. The total number of cancellations is shown in 3(l), which again shows the performance of the dynamic discount pricing strategy surpasses most of the benchmarks. Looking at the benchmark results alone, it can be observed that there are less cancellations when there are more successful shared trips, suggesting conducting shared trips potentially better utilize the fleet of vehicles. However, coincidentally, more cancellations also occur when the platform is generating higher profit, which indicates that the platform may have to make a trade off between short term gain and long term loss.

In Appendix B, we deduce that each cancellation is roughly equivalent to \$4.6 of long-term loss (e.g. based on 10-year customer profit) for the platform. The adjusted total profit is shown in Fig. 3(m). For a profit maximizing platform, it is logical that it adopts the 15% constant discount under the benchmark scenarios. Interestingly, considering long term gains requires the platform with constant discount for shared trips to increase the discount level from 0% to 15%. Under such circumstances, the dynamic discount pricing strategy generates 10.1% more profit, reduces the number of cancellations by 5.8%, and improves the adjusted profit by 26.9%, while conducting 36.6% more shared trips.

Fig. 4 analyses the effects of the dynamic discount pricing strategy on those other than the platform, i.e. passengers and drivers. We first look at the average passenger wait time in Fig. 4(a). It can be observed that the average waiting time is higher with more shared trips conducted. This is intuitive since the second passenger in a shared trip would have to wait longer as the vehicle needs to first pick up another passenger. As the discount pricing strategy produces more shared trips than the benchmarks, the average passenger wait time is therefore higher than the benchmarks (8.8 extra seconds comparing to 15% discount benchmark level). This measure seems to indicate the strategy reduces the passenger's benefit, although the impact of an 8.8 s increase is rather negligible considering the average waiting time of around 4 min. Furthermore, since the cancellations are reduced, it suggests that there are

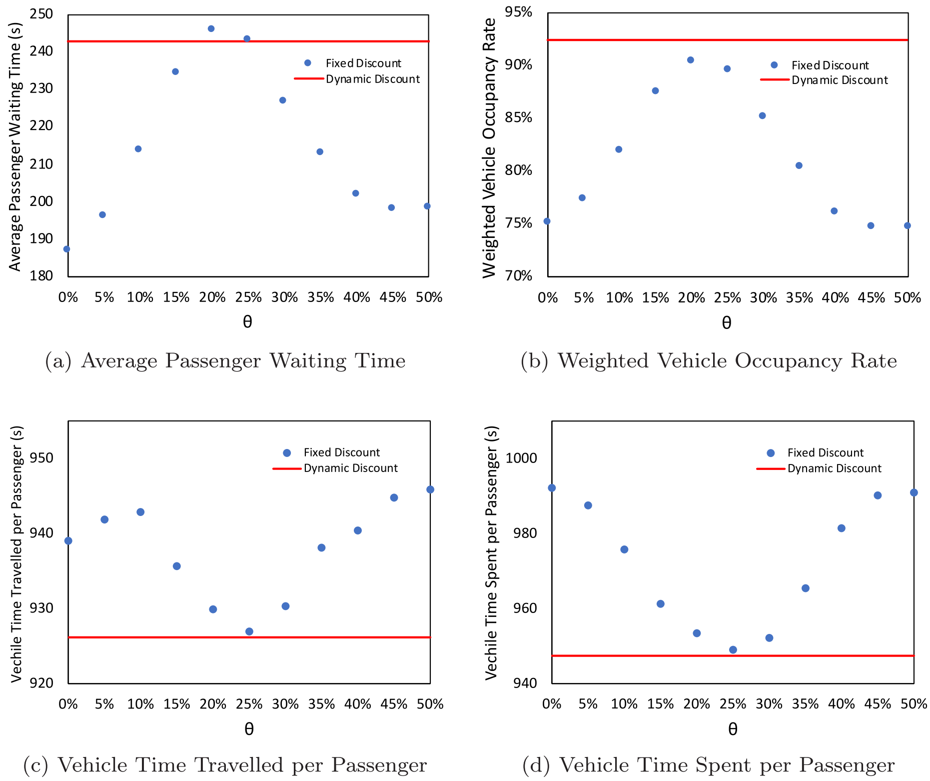


Fig. 4. Effects of the dynamic discount pricing strategy on passengers (average passenger waiting time) and drivers (weighted vehicle occupancy rate, vehicle time travelled/spent per passenger).

passengers serviced who would not have gotten a ride before, and it can be argued that those passengers are better-off under this strategy.

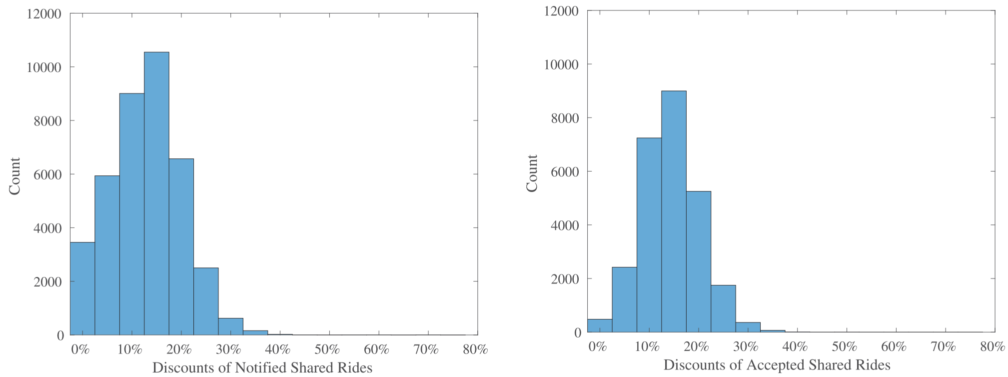
We then look at the weighted vehicle occupancy rate (WVOR) in Fig. 4(b), where the time a vehicle has two passengers on board is weighted twice as when there is only one passenger. The WVOR measures not only vehicle occupancy but also vehicle loading. The dynamic discount pricing strategy increases the WVOR by 5.6% (again, comparing to 15% discount benchmark level), which suggest drivers spend more time with passenger on-board, rather than being idle or during pick-up.

Vehicle time travelled and spent per passenger are shown in Figs. 4(c) and 4(d) respectively. Vehicle time travelled per passenger is the total time a vehicle spent during the Dispatched, Partially Occupied, and Fully Occupied states, divided by the total number of passengers served. Whereas, vehicle time spent per passenger, in addition, includes the time the vehicle spent in Idle states. Both indicators measure how efficient vehicles serve the passengers. For example, if a vehicle can reduce the average time spent per passenger all other things being equal, then the vehicle is able to service more passengers, which is beneficial to all stakeholders. Vehicle time travelled (which exclude idle time) per passenger may be more appropriate if the vehicle parks instead of cruises when idle, since the time vehicles spend on parking does not incur fuel cost to the drivers, and does not cause congestion nor pollution. Furthermore, vehicle time travelled per passenger also provides a sense of the deadheading and detour time. For example, assuming the average trip lengths for passengers serviced are the same, then logically the lower the vehicle time travelled per passenger, the lower the deadheading and detour time, which again shows the efficiency of the system. We distinguish the two because we do not truly know the behaviours of drivers when they drop off the passengers. Regardless, both measures show the dynamic discount strategy outperforms the benchmarks. The average vehicle time travelled/spent per passenger are reduced by 9.5 and 13.8 s respectively. Therefore Figs. 4(b)–4(d) show societal benefits of the strategy as drivers spend more time serving passengers while their efficiency at serving each passenger is also improved.

In summary, by comparing with the benchmark strategy, we show that the adoption of the dynamic discount pricing strategy creates substantial economic benefit for the platform in both the short and long terms, improves the fleet efficiency, and generates positive externalities for the society.

4.3. Properties of dynamic discounts

This section looks at the results produced by the dynamic discount strategy. Fig. 5 shows the distributions of the notified discount levels by the platform, and the discount levels that are accepted by the passengers. We observe that the distribution of notified



(a) Distribution of Notified Discounts (b) Distribution of Accepted Discounts

Fig. 5. Distributions of notified and accepted discounts.

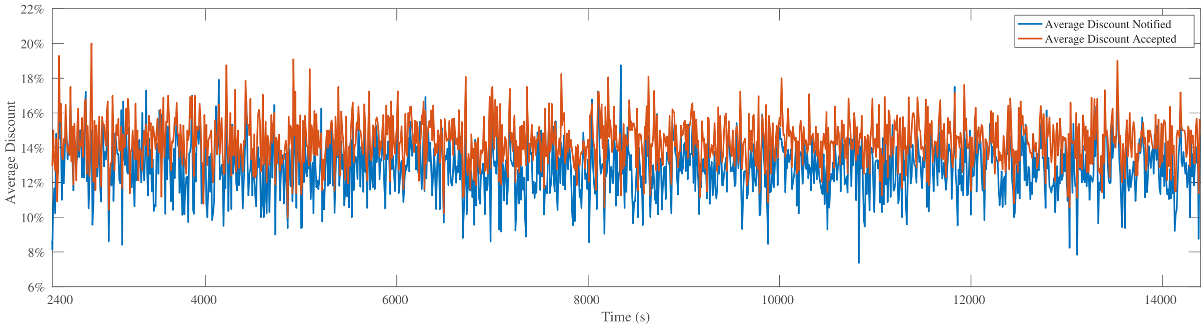


Fig. 6. Average notified and accepted discounts at each matching instance over time. (Note that these two trend lines do not show huge volatility, they generally stay between 8% and 20%.).

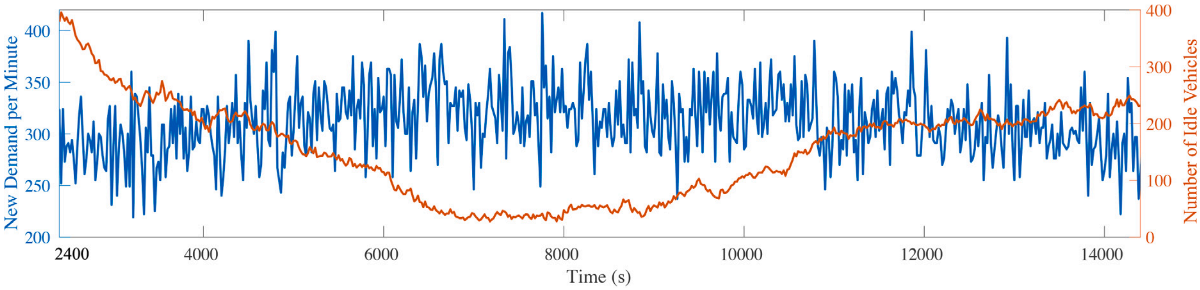
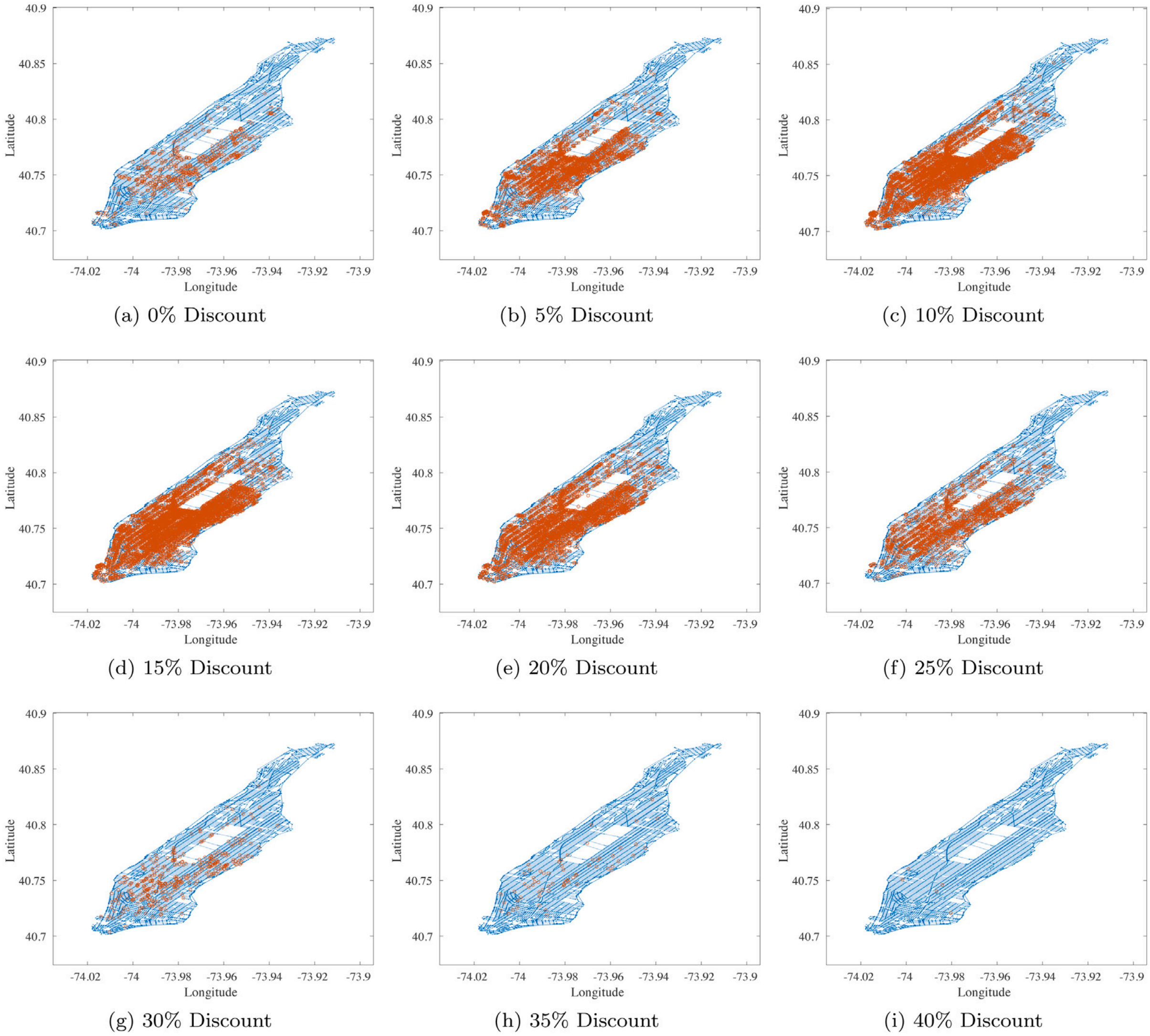


Fig. 7. Rate of new passenger requests and number of idle vehicles over time.

discounts is positively skewed, whereas the distribution of accepted discounts is roughly symmetrical. This can be explained as the passengers would tend to reject lower discounts (especially if they are offered a 0% discount for shared trips) while accept higher discounts. One may ponder why the strategy still offers discount at such low levels given that only a small fraction of the passengers would accept them. We argue that since the strategy aims to maximize expected profit which includes the possibilities of the passenger being in a solo trip or a forced solo trip after declining or accepting the offer, therefore offering passengers higher discounts simply to achieve higher shared trip acceptance rate may not be optimum.

Fig. 6 shows the average notified and accepted discount at each matching instance (10 [s]) over time. We observe that the accepted discount levels are generally higher than the notified discount levels in every matching instance, which is expected due to passengers rejecting shared trips with lower discount. The average notified discount is 12.83% and the average accepted discount is 14.36%. However, under variations in the demand and supply levels as shown in Fig. 7, we cannot observe any long-term patterns or trends of the discount levels that correspond to these changes in market condition. Therefore, it may suggest the strategy is highly individualized and less correlated to macro-level conditions.

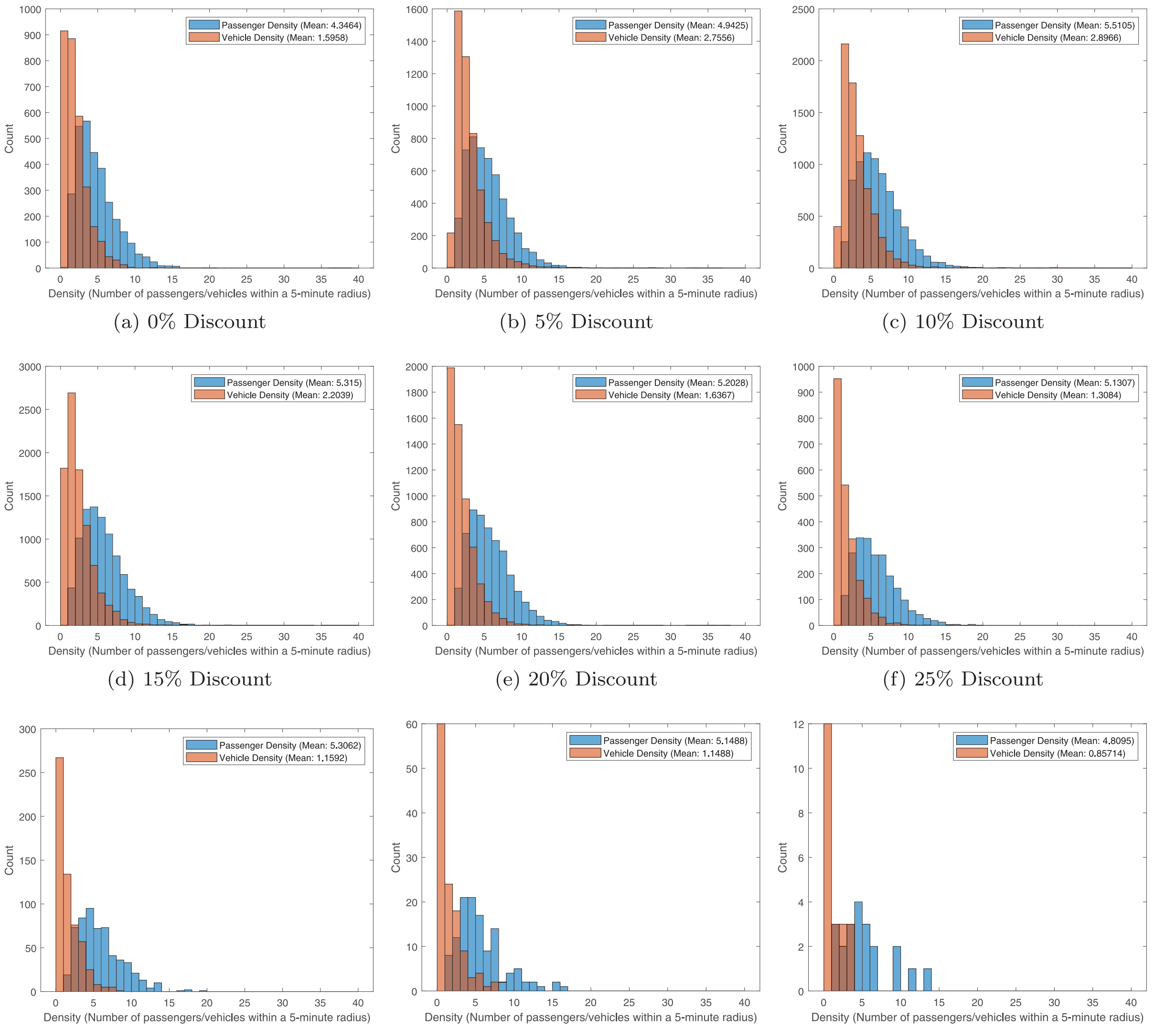


(a) 0% Discount (b) 5% Discount (c) 10% Discount
(d) 15% Discount (e) 20% Discount (f) 25% Discount
(g) 30% Discount (h) 35% Discount (i) 40% Discount

Fig. 8. The origins of passengers who accept the shared trip under the dynamic discount strategy based on the discount level they received.

The spatial orientations of passenger origins of those who accepted shared trips are shown in Fig. 8 based on the discount level they received. We point out two observations from these figures. Firstly, for all passengers that accepted the shared trip, they are mainly orientated in the south-western side of Manhattan where this is more demand. Secondly, although the total number of passengers that accepted different levels of discount varies (see also Fig. 5(b)), the locations the passenger clustering are independent of the discount levels. In other words, we do not observe what one might expect, for example, passengers with lower levels of discount being concentrated in one side of the city where there are lower demand, while passengers with higher levels of discount being concentrated in the other side of the city with higher demand. It is logical that there are more likely to be compatible shared trips in high demand areas, but the platform does not necessarily have to offer higher discount to those passengers to promote shared trips. Since the more compatible the shared trip, the less likely the passengers will reject the shared trip due to smaller amount of inconvenience, and thus the platform does not have to give more discount to compensate. We believe the levels of discount received by the passengers are more dependent on their specific trip details (i.e. the compatibility of each individual pairs' origins and destinations, and order times), which may not be correlated with aggregated demand density.

The imbalance between supply and demand could also have an effect on the discount levels offered to the passengers. For each level of discount offered to the passengers, we show their respective local demand density and supply density in Fig. 9. We define a passenger's local demand/supply density as the number of waiting passengers/idle vehicles within a 5-minute travelling distance at the instance that the passenger is matched. We can observe that the demand densities have roughly the same distributions for all levels of discount. This is consistent with our conjecture from the previous figure, that there is a low correlation between the levels of discount received by the passengers and demand density. Interestingly, the figures show a variation in the supply density



(a) 0% Discount (b) 5% Discount (c) 10% Discount

(d) 15% Discount (e) 20% Discount (f) 25% Discount

(g) 30% Discount (h) 35% Discount (i) 40% Discount

Fig. 9. Passengers' local demand and supply densities distributions at the time they are matched in shared rides for each of discount levels offered.

distributions. With the exception of Fig. 9(a), we can observe that the average supply densities are lower and the distributions are more right-skewed for increasing discount levels. This suggests that passengers who received higher discounts under the strategy is more likely to be in areas of low supply. This is intuitive since a higher discount would imply the passenger is getting a higher compensation likely due to the longer waiting time resulting from the low supply density.

However the average supply density is relatively low in Fig. 9(a), where we would expect it to be the highest if it is consistent with the pattern observed in the other figures. This is to suggest that there is a dis-proportionally high likelihood that a passenger would receive no discount at all even if he/she is in an area of low supply. We hypothesize this is because when a passenger is in a (relatively very) low supply area, the quality of the shared trip for this passenger is likely to be poor, and the strategy would predict there is a low likelihood for the shared trips to be successful even if a high discount is offered. Therefore, it may be better off to offer no discount at all instead of potentially making a loss on the trip.

Finally, we look at joint passenger discounts in Fig. 10. Since we used a grid search approach to solve for the optimal discount (Section 3.3), with a grid size of 21, there are 441 possible discount combinations offered to passenger pairs in shared trips. We plot the number of passenger pairs that are offered each of those 441 possible discount combinations in Fig. 10, where passenger 1 is always the first passenger who boards the shared trip. We observe that the larger bubbles lie close to the line $y = x$, which suggests that the two passengers in one shared trip tend to receive similar levels of discount offered by the platform. There is still a discrepancy between the discount levels offered to the two passengers, the average discount offered to the first passengers is 11.86%, and 13.98% for the second passengers. This discrepancy is intuitive as passenger 2 would always wait longer for pick up than passenger 1, therefore they are compensated more by the dynamic discount strategy.

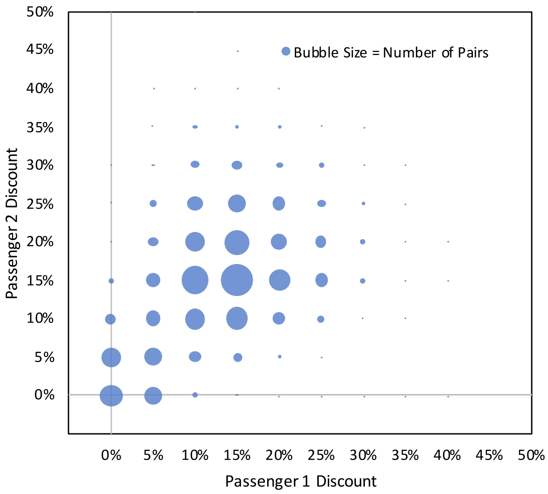


Fig. 10. Joint passenger discount, the bubble size is the number of passenger pairs that are offered the discount levels specified by the x and y axes. Note that passenger 1 is always the first passenger that boards the shared trip (but not always the first one to alight).

5. Summary and future research

This paper has presented a dynamic model of the on-demand mobility market that considers the passengers to be cost and service quality sensitive, impatient, and have a choice to decline services offered by the platform. A dynamic discount pricing strategy personalized for every passenger is proposed for the profit-maximizing platform that promotes ridesharing. The strategy is in the form of a non-linear mixed-integer optimization problem. We transform the problem into two sequential sub-problems that can be solved with limited computational time, and we prove the transformation is equivalent. Numerical experiments show that the adoption of the dynamic discount pricing strategy creates substantial economic benefit for the platform in both the short and long terms, and improves the fleet efficiency and thus generates positive externalities for the society.

The proposed method optimizes considering the current batch of demand and supply. A forward looking algorithm would be advantageous as a future research study. Especially for shared services, as the platform can potentially 'hold' a customer if a high likelihood for a potential pair is imminent within a few matching instances. Relaxing assumptions of this study, such as allowing continuous en-route sharing (where those on a forced solo trip can still be considered to share with another passenger) and fleet size management, may increase the applicability of the strategy. This study focuses on the demand side of the ridesourcing market. However, the supply side of the market also requires attention, as supply can be heterogeneous (e.g. part time and full time drivers) (Ramezani et al., 2022), and drivers do not always accept trips designated by the platform. Therefore, a future study could investigate the manipulation of wages dynamically and strategically to achieve a similar objective — promoting shared rides. How the strategy may adapt to government policies, such as a tax on deadheading or a subsidy for shared trips, and the subsequent effects on social welfare could also be studied.

Another research topic is to consider a ridesharing-promoting strategy that target groups of passengers rather than each individual passenger. Hamedmoghadam et al. (2019) uncover latent collective mobility patterns with specific temporal and spatial properties in different cities. Perhaps this knowledge can be taken advantage of when considering such strategy. We anticipate such strategy would produce vastly different results comparing with the strategy presented in this paper, as the main drivers of such strategy would be macro-level conditions rather than the characteristics of individual trips. Another research direction is to study marketing or operating strategies that promote ridesharing such as selling a bundled shared trip packages to passengers at a discount. Promoting ridesharing to complement public transport is also an interesting topic to be addressed in future works.

CRediT authorship contribution statement

Guipeng Jiao: Conceptualization, Methodology, Software, Validation, Formal analysis, Investigation, Data curation, Writing – original draft, Review & editing. **Mohsen Ramezani:** Conceptualization, Methodology, Validation, Formal analysis, Investigation, Writing – original draft, Review & editing, Supervision, Funding acquisition.

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Appendix A. Expected profit

The expected profit for solo trips used in Eq. (9) is defined explicitly in Eq. (12):

$$E[\pi_{ik}] = E[\mathrm{Pr}_i^{\mathrm{s}}] \cdot \pi_{ik} \tag{12a}$$

where π_{ik} is the deterministic profit of the solo trip as defined in Eq. (8b), and $E[\mathrm{Pr}_i^{\mathrm{s}}]$ is the platform's expected probability of passenger i accepting the solo trip. Assuming the platform is aware of the population βs, the individual $\beta_i s$ are substituted in Eqs. (1a) and (4), therefore, the expected probability is:

$$E[\mathrm{Pr}_i^{\mathrm{s}}] = \frac{e^{(\beta^{\mathrm{s}} - \beta^{\mathrm{t}} w_i^{\mathrm{s}} - \beta^{\mathrm{f}} f_i^{\mathrm{s}})}}{e^{(\beta^{\mathrm{s}} - \beta^{\mathrm{t}} w_i^{\mathrm{s}} - \beta^{\mathrm{f}} f_i^{\mathrm{s}})} + e^{\beta^{\mathrm{o}}}} \tag{12b}$$

Similarly, the expected profit for shared trips is:

$$\begin{aligned} E[\pi_{ijk}^{\mathrm{sq}}] = F(\theta_{i,ijk}^{\mathrm{sq}}, \theta_{j,ijk}^{\mathrm{sq}}) = {} & E[\mathrm{Pr}_i^{\mathrm{h}}] \cdot E[\mathrm{Pr}_j^{\mathrm{h}}] \cdot \pi_{ijk}^{\mathrm{sq}} \\ & + E[\mathrm{Pr}_i^{\mathrm{h}}] \cdot (1 - E[\mathrm{Pr}_j^{\mathrm{h}}]) \cdot (\hat{\pi}_{ik} + E[\pi_j]) \\ & + (1 - E[\mathrm{Pr}_i^{\mathrm{h}}]) \cdot E[\mathrm{Pr}_j^{\mathrm{h}}] \cdot (\hat{\pi}_{jk} + E[\pi_i]) \\ & + (1 - E[\mathrm{Pr}_i^{\mathrm{h}}]) \cdot (1 - E[\mathrm{Pr}_j^{\mathrm{h}}]) \cdot (E[\pi_i] + E[\pi_j]) \end{aligned} \tag{13a}$$

The four terms in Eq. (13a) correspond to the profit made by the platform in four possible cases where (1) both p_i and p_j accept the shared ride, (2) p_i accepts and p_j rejects, (3) p_i rejects and p_j accepts, and (4) both p_i and p_j reject. $E[\mathrm{Pr}_i^{\mathrm{h}}]$ is the platform's expected probability of passenger i accepting the shared trip, and $(1 - E[\mathrm{Pr}_i^{\mathrm{h}}])$ is the platform's expected probability of passenger i not opting for shared trip. Again, the individual $\beta_i s$ and ϵ_i^{w} are substituted by the population βs and ϵ^{w} respectively in Eqs. (1b), (2), and (3) to determine the expected probability of the passengers accepting the shared trip:

$$E[\mathrm{Pr}_i^{\mathrm{h}}] = \frac{e^{(\beta^{\mathrm{h}} - \beta^{\mathrm{t}}(w_i^{\mathrm{h}} + \delta_i) - \beta^{\mathrm{f}} f_i^{\mathrm{s}}(1 - \theta_{i,ijk}^{\mathrm{sq}}))}}{e^{(\beta^{\mathrm{h}} - \beta^{\mathrm{t}}(w_i^{\mathrm{h}} + \delta_i) - \beta^{\mathrm{f}} f_i^{\mathrm{s}}(1 - \theta_{i,ijk}^{\mathrm{sq}}))} + e^{(\beta^{\mathrm{s}} - \beta^{\mathrm{t}} \epsilon^{\mathrm{w}} - \beta^{\mathrm{f}} f_i^{\mathrm{s}})}} \tag{13b}$$

$$E[\mathrm{Pr}_j^{\mathrm{h}}] = \frac{e^{(\beta^{\mathrm{h}} - \beta^{\mathrm{t}}(w_j^{\mathrm{h}} + \delta_j) - \beta^{\mathrm{f}} f_j^{\mathrm{s}}(1 - \theta_{j,ijk}^{\mathrm{sq}}))}}{e^{(\beta^{\mathrm{h}} - \beta^{\mathrm{t}}(w_j^{\mathrm{h}} + \delta_j) - \beta^{\mathrm{f}} f_j^{\mathrm{s}}(1 - \theta_{j,ijk}^{\mathrm{sq}}))} + e^{(\beta^{\mathrm{s}} - \beta^{\mathrm{t}} \epsilon^{\mathrm{w}} - \beta^{\mathrm{f}} f_j^{\mathrm{s}})}} \tag{13c}$$

Furthermore, π_{ijk}^{sq} is the profit made by the platform for the shared trips as defined in Eq. (8a). $\hat{\pi}_{ik}$ and $\hat{\pi}_{jk}$ are the profit made by platform on the forced solo trips with p_i and p_j respectively if their counterpart rejects:

$$\pi_{ijk}^{\mathrm{sq}} = f_i^{\mathrm{s}}(1 - \theta_{i,ijk}^{\mathrm{sq}}) + f_j^{\mathrm{s}}(1 - \theta_{j,ijk}^{\mathrm{sq}}) - c_{ijk}^{\mathrm{sq}} \tag{13d}$$

$$\hat{\pi}_{ik} = f_i^{\mathrm{s}}(1 - \theta_{i,ijk}^{\mathrm{sq}}) - c_{ik} \tag{13e}$$

$$\hat{\pi}_{jk} = f_j^{\mathrm{s}}(1 - \theta_{j,ijk}^{\mathrm{sq}}) - c_{jk} \tag{13f}$$

Additionally, if p_i or p_j rejects the shared trip, the platform could still potentially match them in a solo trip in the future, and thus, we consider the expected future solo trip profit that could be earned. However at the time of the current matching instance, the platform is unknown of the service that could be provided in the future. Therefore, we assume that the future salary cost is based on the position of the driver they are currently matched to. We also assume the probability of a passenger being successfully assigned a solo trip in the future after they rejected a shared trip is a calibrated constant parameter, ξ, which is between 0 and 1. We calibrated this parameter to be 0.65 in our tests. Therefore, the expected future solo trip profits for p_i and p_j are:

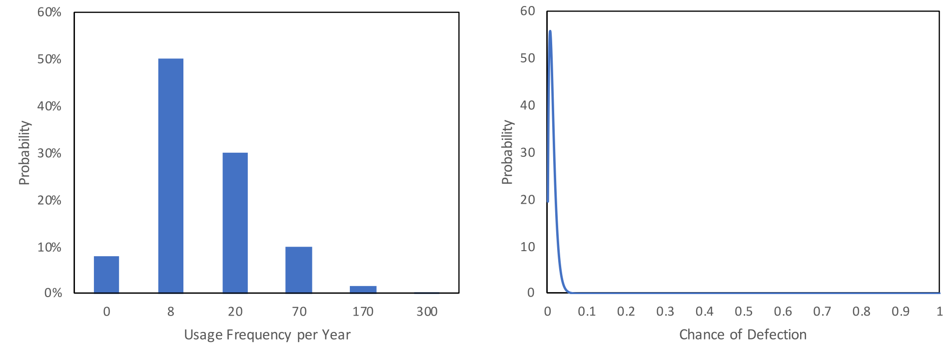
$$E[\pi_i] = \xi(f_i^{\mathrm{s}} - c_{ik}) \tag{13g}$$

$$E[\pi_j] = \xi(f_j^{\mathrm{s}} - c_{jk}) \tag{13h}$$

It should be noted that the probabilities and the profits in each term of Eq. (13a) are highly depended on the discount term θ for the two passengers, which are the decision variables that the proposed discount method optimizes.

Appendix B. Cost of cancellation

For life time value of a customer, we assume that 10 years of future profit are considered. To obtain such value, we assume the distribution of the passengers' yearly usage (Alemi et al., 2019) of the service is shown in Fig. 11(a), where average profit per service is \$2.5 and the cash flows discounted to present value at 8% annually. Furthermore, we assume the probability of defection after a cancellation follows a Beta distribution with $\alpha = 2$ and $\beta = 150$ as shown in Fig. 11(b).



(a) Probability of Yearly usage (b) Probability of Defection

Fig. 11. Passenger probability of yearly usage and probability of defection after cancellation.

We assume the life time value of passenger i, L_i, and the probability of defection after a cancellation Pr_i^d for the same passenger are independent. Then the average cost of cancellation, C, is determined to be \$4.6 as follows:

$$C = E[L_i \times \mathrm{Pr}_i^d] = E[L_i]E[\mathrm{Pr}_i^d] \tag{14a}$$

where:

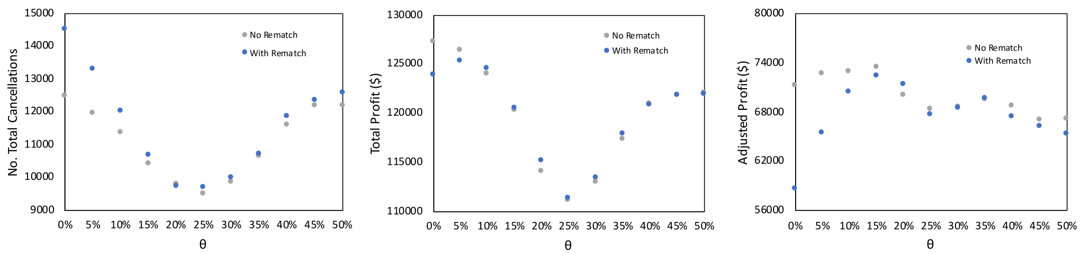
$$E[L_i] = [2.5 \times (0.5 \times 8 + 0.3 \times 20 + 0.1 \times 70 + 0.017 \times 170 + 0.003 \times 300)] \times [\frac{1-(1+8\%)^{-10}}{8\%}] = \$349 \tag{14b}$$

$$E[\mathrm{Pr}_i^d] = \frac{2}{2+150} = 1.32\% \tag{14c}$$

Appendix C. Rematch algorithm

A consequence of allowing passengers to reject is that there will be those who accept the shared trips but their counterpart rejects, which will force them to be in solo trips. We assume that under such scenario, the platform still honours the price discount offered to them, even if the platform makes a loss on the trip. However, after knowing the decision of the passengers, the platform may be able to mitigate a portion of such losses by immediately considering a rematch between the smaller set of passengers who accepted their rides, $\hat{\mathcal{P}}$, and the idle vehicles, $\mathcal{V}$.

Since passengers who accepted and the idle vehicles are considered for rematch, we denote $\hat{\mathcal{P}}_1$ and $\hat{\mathcal{P}}_2$ as those who accepted shared trips and solo trips respectively. Evidently, $\hat{\mathcal{P}}_1$ can be considered in both solo and shared trips whereas $\hat{\mathcal{P}}_2$ can be considered only in solo trips. It is ensured in rematch that the service provided to the passengers will be no worse than their originally offered service. That is, the waiting times and detour times of passengers shall be less or equal to their originally offered trips. (Note that simply honouring the original matching will be a solution that satisfies these constraints, but there could exist a new solution that not only satisfies the constraints but also increases the profit made on this current batch.) Additionally, the passengers pay the trip fare indicated by the initial matching regardless of how they are matched this time. Rematch produces the finalized arrangement, and the platform dispatches vehicles to their designated trips. We found the performance of this algorithm is not significant, and it can produce an adverse effect in certain scenarios. The cancellations, profit, and adjusted profit for benchmarks scenarios with and without rematch algorithm are shown in Fig. 12. The same performance measurement are compared for the strategy with/without rematch, and are shown in Table 5.



(a) Cancellations (b) Profit (c) Adjusted Profit

Fig. 12. Performance measures: total number of cancellations, profit, and adjusted profit, for the benchmarks scenarios with and without the rematch algorithm.

Table 5
Performance measures: total number of cancellations, profit, and adjusted profit, for the proposed dynamic discounting strategy with and without the rematch algorithm.

Performance measurement	With rematch	Without rematch
Total cancellations	9941	9802
Profit ($)	132361.9	132310.9
Adjusted profit ($)	92597.9	93102.9

We observe that the benchmark scenarios generally have slightly poor performances with the rematch algorithm comparing to without rematch. The decline in performance is more noticeable when the constant discounts are at the two ends of the spectrum. We suspect that it is due to the myopic nature of the algorithm; rematch may improve the performances of a single instance, however it may have deteriorated the matching efficiencies for the future non-explicitly. The strategy with rematch only produces marginally better profit comparing to without rematch, while the total cancellations are increased and adjusted profit is reduced. Although the discrepancies are not as significant as observed for the benchmark scenarios. Due to these observations, we decided to exclude the rematch algorithm from the main body of this study.

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