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A real-time cooperation mechanism in duopoly e-hailing markets

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ABSTRACT

In a competitive e-hailing market, each participating platform can only utilize a portion of the total demand (passengers) and supply (drivers), in which this fragmentation leads to market inefficiencies. Therefore in this study, we propose a decentralized dynamic cooperation mechanism, inspired by capacity-sharing strategies, between two platforms to mitigate market fragmentation. With the proposed mechanism, the platforms can offer to (i) refer their passengers and (ii) temporarily (for a single trip) lease idle vehicles to the other platform, at varying optimized prices. In its essence, whenever a platform is under-supplied while the other platform is over-supplied in a localized area, the proposed mechanism could help the platforms bridge their supply and demand. The proposed mechanism is constructed as an optimization problem incorporated in the batch-matching algorithm. We test the proposed cooperation mechanism in a disaggregated dynamic model. We consider multiple scenarios, which include symmetrical and asymmetrical duopolies. The performance of the proposed cooperation mechanism is compared to equivalent non-cooperative duopolies and equivalent monopolies. We show that the proposed cooperation mechanism leads to simultaneous improvements in all performance indicators for all stakeholders (passenger pickup time, passenger cancellations, driver income, and platform profitability) compared to equivalent non-cooperative duopolies. Additionally, the proposed cooperation mechanism is shown to be especially effective as the asymmetry between the duopoly increases.

1. Introduction

In the e-hailing market, Transport Network Companies (TNCs) provide digital platforms to passengers who need a ride and drivers who want to provide service. The passengers make trip requests in real-time, and the platforms need to match them to available drivers on demand. This real-time matching nature means the platforms may benefit from network effects (Wu et al., 2020) on both the demand (passengers) and supply (drivers) sides, as more optimal matches can be established with more active passengers and/or vehicles. However, when there are more than one platform in the e-hailing market, passengers and drivers may opt to use one platform over the others. Hence, each platform can only utilize a portion of the total demand and supply. In other words, the market is fragmented. A fragmented market dilutes the network effect (Zhang et al., 2019) which can be otherwise enjoyed by a single platform that has access to all the resources. Additionally, it may worsen the spatial and temporal imbalance between supply and demand. Therefore, for a given (exogenous) potential passenger demand and driver supply in the e-hailing market, the operational efficiency would be reduced with the increasing number of platforms. Consequently, it may lead to increased passenger pickup time, more passenger cancellations, reduced platform profitability, and lowered driver income. In this study, we propose a dynamic cooperation mechanism between two platforms to mitigate these inefficiencies.

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A number of studies have investigated the inefficiencies due to the fragmentation in the multi-platform e-hailing market. Séjourné et al. (2018) considered the case where the demand is exogenously split between multiple platforms. They defined the price of fragmentation as the increase in supply rebalancing cost incurred by the platforms, compared to the cost of a single platform serving the aggregate demand. They found that the additional cost due to fragmentation either vanishes or grows unbounded depending on the nature of the exogenous demand. Similarly, Kondor et al. (2022) measured the cost of market fragmentation or the cost of non-coordination as the additional number of vehicles needed when the platforms only have partial shares of the demand. They showed that each additional operator in the market can increase the total number of vehicles needed by up to 67%. Zhou et al. (2022a) on the other hand took a different approach. They considered the Nash equilibrium solutions of the competitive ride-sourcing market, where no platform can improve its profitability by unilaterally changing its strategy. They found that as the number of platforms increases, the key market measures (such as consumer surplus, platform profits, and social welfare) displayed diverse trends of changes. These trends are depended on whether the on-demand matching between passengers and drivers exhibits increasing, constant, or decreasing returns to scale.

A monopoly is exempt from the inefficiencies due to fragmentation, however, it may not be a desired alternative to a multi-platform e-hailing market. Based on standard economic models, a single monopoly creates a deadweight loss which indicates the market is inefficient (Harberger, 1954). Moreover, a monopoly can also engage in price discrimination to extract consumer surplus. Therefore, it is more desirable to design a mechanism that could address the inefficiencies of fragmentation in the multi-platform e-hailing market. Bao et al. (2022) and Zhou et al. (2022b) considered a third-party integrator, while (Cohen and Zhang, 2022) considered introducing a new joint service between competitors. All of these strategies were reported to improve social welfare. Inspired by the capacity-sharing strategy (e.g., Li and Zhang, 2015), we propose a novel *'referral'* cooperation mechanism between two competitive platforms in the e-hailing market.

Under capacity-sharing strategies, companies with insufficient capacity may borrow (and may pay for) excess capacity from other companies. Capacity-sharing strategies are common in many industries. Some examples of capacity-sharing strategies include: code-sharing in the airline industry (Hu et al., 2013), inventory trade in the retail industry (Çömez et al., 2012), capacity reservation between shipping forwarders (Guo and Wu, 2018), utilization of dedicated bus lanes for ride-hailing services (Fayed et al., 2023), etc. Capacity-sharing strategies are popular as they are effective in addressing a mismatch between supply and demand. Notably, the spatial and temporal imbalance between supply and demand is commonly recognized in the e-hailing market (Zha et al., 2018; Zhu et al., 2021; Chen et al., 2021b; Li et al., 2021; Beojone and Geroliminis, 2023b; Zhu et al., 2023; Valadkhani and Ramezani, 2023). Hence, we introduce the capacity-sharing strategy to the e-hailing market. In its essence, whenever a platform is under-supplied while the other platform is over-supplied in a localized area, the mechanism should help the platforms to bridge their supply and demand.

More specifically, the mechanism is formulated in a 'referral' approach. With the proposed cooperation mechanism, the platforms have the option to refer their passengers to the other platform (i.e. offering excess capacity). When a platform intends to refer a passenger, it asks and pays (less than what is originally charged from the passenger to be profitable) the other platform to service the passenger. The platforms are driven by their objectives of profit maximization. Therefore, the platform may choose to exercise the option and refer a passenger if it is economically optimal, for example when there are no nearby idle vehicles. Furthermore, the receiving platform can choose not to accept the referral if it is not profitable. Similarly, suppose a platform finds one of its idle vehicles to be far away from any of the passenger requests. In that case, the platform can offer to temporarily (for a single trip) lease this idle vehicle to the other platform. This proposed cooperation mechanism requires minimal changes to the existing platform operations, and the passengers and drivers need not be aware of the mechanism. In practice, firms may have different objectives. For example, at one extreme, a firm may wish to eliminate its competition, and a corresponding strategy may be to engage in price wars, which defeat the purpose of a cooperation strategy. In this study, we wish to avoid speculating on each platform's intentions, and we assume the platforms are purely short-term profit driven. Additionally, the proposed cooperation mechanism allows both companies to optimize for themselves, rather than a central integrator that dictates how they should cooperate. Therefore, it should encourage cooperation even when there is a dominant firm. Correspondingly, this proposed cooperation mechanism treats both platforms fairly regardless of the size and dominance of each platform. It only requires the participating platforms to act rationally to maximize their own short-term profit, and not engage in activities with the goal of sabotaging the other party.

We utilize a disaggregated dynamic model to test the performance of the dynamic cooperation mechanism. We consider multiple scenarios, which include symmetrical and asymmetrical duopolies. In this study, we assume platforms in asymmetrical duopolies are differentiated by their respective operational fleet sizes and potential passenger demands, ceteris paribus. Whereas platforms in symmetrical duopolies have identical operational fleet sizes and potential passenger demands. The performance of the proposed cooperation mechanism is compared to equivalent standard duopolies and equivalent monopolies. In the disaggregated dynamic model, we assume the passengers choose one of the platforms based on given probabilities upon entering the market. They are also assumed to be impatient and service quality sensitive, i.e. they may cancel and leave the platform if they are not matched by the platform in a timely manner (Xu et al., 2022), or if they are not satisfied with the match established by the platform (Wang et al., 2019). We consider the passengers to be heterogeneous who have different levels of patience and values of time. The passengers also have varying origins and destinations, along with different times for when they enter the market. We use real passenger demand data (New York City yellow taxi trip record) for these trip characteristics. The platforms are considered to adopt batch matching algorithms, where the waiting passengers and the idle vehicles are accumulated and matched by the platforms periodically over time. The cooperation mechanism is integrated with the platforms' matching algorithm. During each platform's matching instance, the algorithm maximizes the expected profit of the current batch, considering the cost of cancellation as well as the cost due to deadheading. The output of algorithm simultaneously decides (i) which of its own passengers and vehicles should be matched, (ii) which of its own passengers and vehicles should be offered for referral and lease to the other platform, and at what price, and (iii) which of the referred passengers and leasable vehicles from the other platform should be accepted and utilized.

The remainder of the paper is structured as follows. In Section 2, we present the disaggregated dynamic model of the e-hailing market, and elaborate on the interactions among all market participants. In Section 3, we formulate the cooperation mechanism. In Section 4, we present and discuss the results. Finally, in Section 5, we conclude and summarize the study.

2. Dynamic model of the e-hailing market

2.1. The passengers

In the proposed disaggregated dynamic model, we consider the passengers to be heterogeneous. When there is more than one platform in the market, we assume that each passenger uses only one platform based on given probabilities without multi-homing. The passengers join the platform through their mobile applications and specify their origins and destinations. Immediately after, the platform would attempt to match the passenger (see Sections 2.4 and 3), and indicate to the passengers that matching is in progress. The passengers would wait to be matched. Consider the case of a duopoly, we denote the set of all passengers waiting to be matched who joined platform 1 and 2 as $\mathcal{P}_1 = \{p_{1,1}, \dots, p_{m,1}\}$ and $\mathcal{P}_2 = \{p_{1,2}, \dots, p_{n,2}\}$, respectively. An individual waiting passenger in platform 1 is denoted as $p_{i,1} \in \mathcal{P}_1$, and an individual waiting passenger in platform 2 is denoted as $p_{j,2} \in \mathcal{P}_2$.

The passengers are impatient when they are waiting to be matched. If they are not matched by the platform within a certain time frame, they will cancel their orders and use other modes of travel such as public transport (Type I cancellation). Consider passenger $p_{i,1}$, we denote his/her match waiting patience as $\bar{m}_{i,1}$. The passengers are also sensitive to service quality and cost. When the platform matches a passenger, $p_{i,1}$, to a vehicle, $v_{a,1}$, the passenger will be notified and presented with the trip details on their mobile application. The trip details include the exact waiting time to be picked up, $w_{i,1}^{a,1}$, and fare price, $f_{i,1}$. The passenger then has the option to cancel the trip if they are not satisfied with the service offered by the platform (Type II cancellation). In practice, some platforms impose penalties on passengers who cancel their trips after being matched. In this study, we assume there is no such penalty for passengers associated with this cancellation. We use utility-based choice modeling to predict the passenger's behavior in accepting the offered trip by the platform. We assume that the passenger perceives the utility of the trip offered and all other modes of travel as follows:

$$\text{Trip offered}: \ u_{i,1}^{\mathrm{s}} = \beta_{i,1}^{\mathrm{s}} - \beta_{i,1}^{\mathrm{w}} w_{i,1}^{a,1} - \beta_{i,1}^{\mathrm{f}} (f_{i,1} - f_{i,1}^{\mathrm{f}}) + \epsilon_{i,1}^{\mathrm{s}} \tag{1}$$

$$\text{Other modes}: \ u_{i,1}^{\mathrm{o}} = u_{\mathrm{o}} + \epsilon_{i,1}^{\mathrm{o}}, \tag{2}$$

where $\beta_{i,1}^{\mathrm{s}}$ and u_{o} are the utility constants of the two options, while $\beta_{i,1}^{\mathrm{w}}$ and $\beta_{i,1}^{\mathrm{f}}$ are the per unit time and per unit price utility coefficients for passenger $p_{i,1}$, respectively. $\epsilon_{i,1}^{\mathrm{s}}$ and $\epsilon_{i,1}^{\mathrm{o}}$ are random error terms for the two choices, which each is assumed to be identically and independently distributed with a Gumbel distribution. In this study, we do not consider pricing strategies (which have been thoroughly investigated by the literature, e.g. Wang et al. (2016)), whereas we considered a defined price structure and fixed pricing parameters (over time for a platform in an experiment) for trip fares (See Section 2.3). Without loss of generality, we assume there is a 'fair' price ($f_{i,1}^{\mathrm{f}}$) for each passenger based on their trip length, and they may gain or lose utility depending on the actual price asked by the platform. Then the probability of the passenger accepting the trip offered by the platform, $\mathrm{Pr}_{i,1}^{\mathrm{s}}$, can be calculated:

$$\mathrm{Pr}_{i,1}^{\mathrm{s}} = \frac{e^{\theta u_{i,1}^{\mathrm{s}}}}{e^{\theta u_{i,1}^{\mathrm{s}}} + e^{\theta u_{i,1}^{\mathrm{o}}}}, \tag{3}$$

where θ is the scale factor, which is assumed to be 1 in this study. After accepting the trip, the passenger will be stationary at their origin to wait for the dispatched vehicle for pick up, and they will leave the platform once they have arrived at their destination.

2.2. The vehicles

Numerous studies focused on the nuances of drivers' choices in the e-hailing market (e.g., Ramezani et al., 2022; Fielbaum and Tirachini, 2021; Jian et al., 2016). However, in this paper, we adopt a more straightforward approach to model drivers' behavior. In this study, we assume that the platforms have full control of their fleet of vehicles. That is, the drivers will always accept the trips designated by the platforms, and they follow the routes directed by the platform, which is the shortest-time path between any two locations. The vehicles are also assumed to be stationary (see Gao et al. (2022), also the alternative, vehicle repositioning, has been studied extensively, e.g. Syed et al. (2021)) after the completion of each trip as the drivers wait to be matched with the next passenger by the platform. Therefore, the three states that a vehicle could be in are: idle, matched (dispatched but unoccupied), and occupied. The idle vehicles are considered for matching by the platform. We denote a single idle vehicle from the set of all idle vehicles working in platforms 1 and 2 as $v_{a,1} \in \mathcal{V}_1 = \{v_{1,1}, \dots, v_{k,1}\}$ and $v_{b,2} \in \mathcal{V}_2 = \{v_{1,2}, \dots, v_{l,2}\}$, respectively. Once a vehicle is matched, and the corresponding passenger accepts the trip, then the vehicle is dispatched to pick up the passenger. As the vehicle arrives at the location of the passenger, the vehicle becomes occupied and heads for the passenger's destination. Drivers are paid based on the time they spend in the matched and occupied states, we detail their wage structure in Section 2.3.

2.3. The platforms

The platforms are aware of the origins and destinations of their own waiting passengers, as well as the locations of their idle vehicles. Let us consider a waiting passenger $p_{i,1} \in \mathcal{P}_1$, and an idle vehicle $v_{a,1} \in \mathcal{V}_1$. We denote the origin and destination of $p_{i,1}$, and the position of $v_{a,1}$ as $O_{i,1}$, $D_{i,1}$, and $V_{a,1}$, respectively. Furthermore, the travel time of the shortest path between any two locations, such as from $O_{i,1}$ to $D_{i,1}$, is denoted as $|O_{i,1}D_{i,1}|$.

We assume the fare structure for both platforms consists of a fixed base price (λ_1) and a trip duration-dependent variable price (the price per unit time is λ_2). Note that these two parameters will remain constant for a platform during the span of an experiment, though it might be different for each platform under different experiment settings. Using platform 1 as an example, it is shown as follows:

$$f_{i,1} = \lambda_{1,1} + \lambda_{2,1}|O_{i,1}D_{i,1}|. \tag{4}$$

Note that for the 'fare' price introduced in Eq. (1), we denote the 'fare' price parameters as λ_1^{f} and λ_2^{f} respectively.

In this study, we assume that drivers are paid for any time they spend en-route, which includes the states they spent matched and occupied (see for instance Jiao and Ramezani, 2022). We assume the payment per unit time driver spent matched and occupied are λ_3 and λ_4, respectively. Therefore, if Platform 1 dispatches $v_{a,1}$ to pick up $p_{i,1}$ given that the passenger has accepted the trip, then the wage to be paid, $c_{i,1}^{a,1}$, and the profit made by Platform 1, $\pi_{i,1}^{a,1}$, can be determined respectively as follows:

$$c_{i,1}^{a,1} = \lambda_{3,1}|V_{a,1}O_{i,1}| + \lambda_{4,1}|O_{i,1}D_{i,1}| \tag{5}$$

$$\pi_{i,1}^{a,1} = f_{i,1} - c_{i,1}^{a,1}. \tag{6}$$

There are two reasons for the adoption of this wage structure. Firstly, since we assume the platforms have full control of their fleet and the drivers always accept the designated trips, then it would be unfair to dispatch a vehicle to a far away passenger without paying for the empty miles traveled. Secondly, since the empty miles have to be paid for (i.e., penalized), it would be illogical to dispatch vehicles to passengers that are far away. Therefore, if a platform aims to maximize profit, this wage structure non-explicitly lowers the pickup time and curbs the wild goose chase (WGC) problem.

2.4. Matching algorithms

We assume that the platforms utilize a batch matching algorithm, which is used in practice by platforms such as Didi, and considered by a number of studies (e.g. Chen et al., 2021a; Yang and Ramezani, 2022; Tafreshian and Masoud, 2020; Alisoltani et al., 2022; Qin et al., 2021). With a batch matching algorithm, the idle vehicles and waiting passengers are accumulated and matched by each of the platforms every Δ seconds. There are many ways a platform can match the passengers and the vehicles depending on the platform's objectives. This Section demonstrates a standard myopic profit-maximizing algorithm. The actual matching algorithms used in this study will be introduced later in Section 3, however, the standard myopic profit-maximizing algorithm here is the foundation upon which the latter algorithms are built. Consider the given sets $\mathcal{P}_1$ and $\mathcal{V}_1$ for platform 1. Let $\mathcal{E}_1$ be the set of edges connecting each element of $\mathcal{P}_1$ and $\mathcal{V}_1$, where an edge $(p_{i,1}, v_{a,1}) \in \mathcal{E}_1$ connects $p_{i,1} \in \mathcal{P}_1$ and $v_{a,1} \in \mathcal{V}_1$. The standard myopic profit-maximizing matching algorithm can be formulated as follows:

$$\max_{x_{i,1}^{a,1}} \sum_{(p_{i,1}, v_{a,1}) \in \mathcal{E}_1} \pi_{i,1}^{a,1} x_{i,1}^{a,1} \tag{7}$$

$$\text{s.t.} \quad \sum_{a=1}^{k} x_{i,1}^{a,1} \leq 1 \quad \forall\, p_{i,1} \in \mathcal{P}_1 \tag{8}$$

$$\sum_{i=1}^{m} x_{i,1}^{a,1} \leq 1 \quad \forall\, v_{a,1} \in \mathcal{V}_1 \tag{9}$$

$$x_{i,1}^{a,1} \in \{0,1\} \quad \forall\, p_{i,1} \in \mathcal{P}_1,\ v_{a,1} \in \mathcal{V}_1 \tag{10}$$

In Eq. (7), $x_{i,1}^{a,1}$ is the binary decision variable, with $x_{i,1}^{a,1} = 1$ indicating that designating $v_{a,1}$ to pick up $p_{i,1}$ is in the optimal solution. Eqs. (8) and (9) are constraints that ensure each passenger and each vehicle is matched at most once. After each execution of the algorithm, the platform then notifies the corresponding matched passengers of the trip details (Section 2.1). It is after the passengers' acceptance, that the vehicles shall be dispatched to deliver the service.

3. The cooperation mechanism: Passenger and vehicle referral

3.1. Formulation

To address the inefficiencies due to market fragmentation, we propose a referral cooperation mechanism inspired by the capacity-sharing strategies. Note that for brevity, the mechanism is formulated from the perspective of platform 1 in this section, though platform 2 follows the same procedures.

The mechanism is integrated with the disaggregated dynamic model and works as follows. After platform 1 receives a trip request from passenger $p_{i,1}$, then the platform has the option to refer this passenger to platform 2. The platform may intend to exercise this option to maximize profit, for example, if there are no close-by vehicles to $p_{i,1}$, then it might be more beneficial to let platform 2 provide the service. Platform 1 would still charge the passenger their trip fare $f_{i,1}$, then upon successful passenger referral, it would make a payment, $f^{\mathrm{r}}_{i,1}$, to platform 2 to service this passenger, thus making a profit of $\pi^{\mathrm{r}}_{i,1} = f_{i,1} - f^{\mathrm{r}}_{i,1}$. From platform 2's perspective, we denote the passenger offered for referral as $p_{\hat{i},\hat{1}}$, and the price associated as $f^{\mathrm{r}}_{\hat{i},\hat{1}}$ (note $f^{\mathrm{r}}_{\hat{i},\hat{1}} = f^{\mathrm{r}}_{i,1}$). Platform 2 would determine if it is economically optimum to service this passenger. If indeed it is and vehicle $v_{b,2}$ is dispatched, then platform 2 makes a profit of $\pi^{b,2}_{\hat{i},\hat{1}} = f^{\mathrm{r}}_{\hat{i},\hat{1}} - c^{b,2}_{\hat{i},\hat{1}}$.

The decisions for the referrals (both the referrer's offer and the recipient's acceptance/rejection) are determined during the respective platform's matching. We assume the two platforms' matching instances are staggered. That is, after platform 1's initial matching and referral, the decision from platform 2 regarding the referrals would be made before platform 1's next matching instance, and vice versa. Therefore, if platform 2 declines (accepts) to service $p_{\hat{i},\hat{1}}$, then platform 1 will (will not) include $p_{i,1}$ in the next matching instance.

A similar operation can be applied to the drivers. Platform 1 has the option to offer any of its vehicles for lease (short-term for only one ride) to platform 2, and vice versa. Using a vehicle from platform 1, $v_{a,1}$, as an example, when platform 2 receives the lease offer, we denote the vehicle as $v_{\hat{a},\hat{1}}$. If platform 2 wishes to utilize this vehicle to service one of its passengers, $p_{j,2}$, then it not only has to pay for the driver's en-route wages $c^{\hat{a},\hat{1}}_{j,2}$, but also an extra leasing fee, $c^{\mathrm{r}}_{\hat{a},\hat{1}}$, set by platform 1. Therefore, upon a successful lease, platform 1 makes a profit of $\pi^{\mathrm{r}}_{a,1} = c^{\mathrm{r}}_{a,1}$ (again, $c^{\mathrm{r}}_{a,1} = c^{\mathrm{r}}_{\hat{a},\hat{1}}$), whereas platform 2 would make a profit of $\pi^{\hat{a},\hat{1}}_{j,2} = f_{j,2} - c^{\hat{a},\hat{1}}_{j,2} - c^{\mathrm{r}}_{\hat{a},\hat{1}}$. The leased vehicle is returned to platform 1 once the passenger is dropped off. Similarly, if platform 2 decides not to use the vehicle, then platform 1 will be informed prior to its next matching instance.

Formally, we consider the following sets. The waiting passengers from platform 1, $p_{i,1} \in \mathcal{P}_1 = \{p_{1,1}, \dots, p_{m,1}\}$; the waiting passengers that platform 1 has referred to platform 2, $p_{\hat{i},\hat{1}} \in \mathcal{P}_{\hat{1}} = \left\{p_{1,\hat{1}}, \dots, p_{\hat{m},\hat{1}}\right\}$; the referral prices associated with each of the referred passengers, $f^{\mathrm{r}}_{\hat{i},\hat{1}} \in \mathcal{F}_{\hat{1}} = \left\{f^{\mathrm{r}}_{1,\hat{1}}, \dots, f^{\mathrm{r}}_{\hat{m},\hat{1}}\right\}$; the idle vehicles from platform 1, $v_{a,1} \in \mathcal{V}_1 = \{v_{1,1}, \dots, v_{k,1}\}$; the idle vehicles platform 1 offered to lease to platform 2, $v_{\hat{a},\hat{1}} \in \mathcal{V}_{\hat{1}} = \left\{v_{1,\hat{1}}, \dots, v_{\hat{k},\hat{1}}\right\}$, and the lease prices associated with these vehicles, $c^{\mathrm{r}}_{\hat{a},\hat{1}} \in \mathcal{C}_{\hat{1}} = \left\{c^{\mathrm{r}}_{1,\hat{1}}, \dots, c^{\mathrm{r}}_{\hat{k},\hat{1}}\right\}$. Note that at any time, $P_1 \cap P_{\hat{1}} = \emptyset$ and $V_1 \cap V_{\hat{1}} = \emptyset$, that is a passenger (vehicle) cannot simultaneously be considered for matching by both platforms. Similarly we have the followings sets for platform 2: $p_{j,2} \in \mathcal{P}_2 = \{p_{1,2}, \dots, p_{n,2}\}$, $p_{\hat{j},\hat{2}} \in \mathcal{P}_{\hat{2}} = \left\{p_{1,\hat{2}}, \dots, p_{\hat{n},\hat{2}}\right\}$, $f^{\mathrm{r}}_{\hat{j},\hat{2}} \in \mathcal{F}_{\hat{2}} = \left\{f^{\mathrm{r}}_{1,\hat{2}}, \dots, f^{\mathrm{r}}_{\hat{n},\hat{2}}\right\}$, $v_{b,2} \in \mathcal{V}_2 = \{v_{1,2}, \dots, v_{l,2}\}$, $v_{\hat{b},\hat{2}} \in \mathcal{V}_{\hat{2}} = \left\{v_{1,\hat{2}}, \dots, v_{\hat{l},\hat{2}}\right\}$, and $c^{\mathrm{r}}_{\hat{b},\hat{2}} \in \mathcal{C}_{\hat{2}} = \left\{c^{\mathrm{r}}_{1,\hat{2}}, \dots, c^{\mathrm{r}}_{\hat{l},\hat{2}}\right\}$.

The cooperation mechanism is incorporated in the matching algorithm, from the perspective of platform 1, it can be formulated as follows. Let $\mathcal{E}_1$ be the set of edges connecting each element from $\mathcal{P}_1$ and $\mathcal{V}_1$, $\bar{\mathcal{E}}_1$ be the set of edges connecting each element from $\mathcal{P}_{\hat{2}}$ and $\mathcal{V}_1$, and $\tilde{\mathcal{E}}_1$ be the set of edges connecting each element from $\mathcal{P}_1$ and $\mathcal{V}_{\hat{2}}$. The profit maximizing matching and the cooperation mechanism are simultaneously determined through the following optimization (the bipartite graph is visualized in Fig. 1):

$$\max_{x^{a,1}_{i,1}, x^{a,1}_{\hat{j},\hat{2}}, x^{\hat{b},\hat{2}}_{i,1}, x^{\mathrm{r}}_{i,1}, x^{\mathrm{r}}_{a,1}, f^{\mathrm{r}}_{i,1}, c^{\mathrm{r}}_{a,1}} \quad \sum_{\mathcal{E}_1} \pi^{a,1}_{i,1} x^{a,1}_{i,1} + \sum_{\bar{\mathcal{E}}_1} \pi^{a,1}_{\hat{j},\hat{2}} x^{a,1}_{\hat{j},\hat{2}} + \sum_{\tilde{\mathcal{E}}_1} \pi^{\hat{b},\hat{2}}_{i,1} x^{\hat{b},\hat{2}}_{i,1} + \sum_{i=1}^{m} E[\pi^{\mathrm{r}}_{i,1}] x^{\mathrm{r}}_{i,1} + \sum_{a=1}^{k} E[\pi^{\mathrm{r}}_{a,1}] x^{\mathrm{r}}_{a,1} \tag{11}$$

$$\text{s.t.} \quad \sum_{a=1}^{k} x^{a,1}_{i,1} + \sum_{\hat{b}=1}^{\hat{l}} x^{\hat{b},\hat{2}}_{i,1} + x^{\mathrm{r}}_{i,1} = 1 \quad \forall\, p_{i,1} \in \mathcal{P}_1 \tag{12}$$

$$\sum_{a=1}^{k} x^{a,1}_{\hat{j},\hat{2}} \leq 1 \quad \forall\, p_{\hat{j},\hat{2}} \in \mathcal{P}_{\hat{2}} \tag{13}$$

$$\sum_{i=1}^{m} x^{a,1}_{i,1} + \sum_{\hat{j}=1}^{\hat{n}} x^{a,1}_{\hat{j},\hat{2}} + x^{\mathrm{r}}_{a,1} = 1 \quad \forall\, v_{a,1} \in \mathcal{V}_1 \tag{14}$$

$$\sum_{i=1}^{m} x^{\hat{b},\hat{2}}_{i,1} \leq 1 \quad \forall\, v_{\hat{b},\hat{2}} \in \mathcal{V}_{\hat{2}} \tag{15}$$

$$x^{a,1}_{i,1}, x^{a,1}_{\hat{j},\hat{2}}, x^{\hat{b},\hat{2}}_{i,1}, x^{\mathrm{r}}_{i,1}, x^{\mathrm{r}}_{a,1} \in \{0,1\} \tag{16}$$

Eq. (11) maximizes the expected profit to be generated from the current matching instance. The first term $\sum_{\mathcal{E}_1} \pi^{a,1}_{i,1} x^{a,1}_{i,1}$ represents the total profit to be made from the matches between platform 1's own passengers and vehicles. The terms $\sum_{\bar{\mathcal{E}}_1} \pi^{a,1}_{\hat{j},\hat{2}} x^{a,1}_{\hat{j},\hat{2}}$ and $\sum_{\tilde{\mathcal{E}}_1} \pi^{\hat{b},\hat{2}}_{i,1} x^{\hat{b},\hat{2}}_{i,1}$ represent the total profit to be made by platform 1 from matching its own vehicles to platform 2's referred passengers, and matching its own passengers to platform 2's leasable vehicles, respectively. While $\sum_{i=1}^{m} E[\pi^{\mathrm{r}}_{i,1}] x^{\mathrm{r}}_{i,1}$ and $\sum_{a=1}^{k} E[\pi^{\mathrm{r}}_{a,1}] x^{\mathrm{r}}_{a,1}$ are the expected profit to be made by platform 1 by offering to refer its passengers and lease its vehicles, respectively.

Eq. (12) is the constraint that ensures each of platform 1's own passengers will be matched to at most one vehicle, and if they are not matched then they will be referred to platform 2. Note that the option for not referring nor matching an own passenger, i.e., 'holding' this passenger now to match them in the future, is not considered, since the option to refer can always generate a higher expected profit than to 'hold'. This conjecture is further explained in Section 3.3. Eq. (13) is the constraint that dictates each

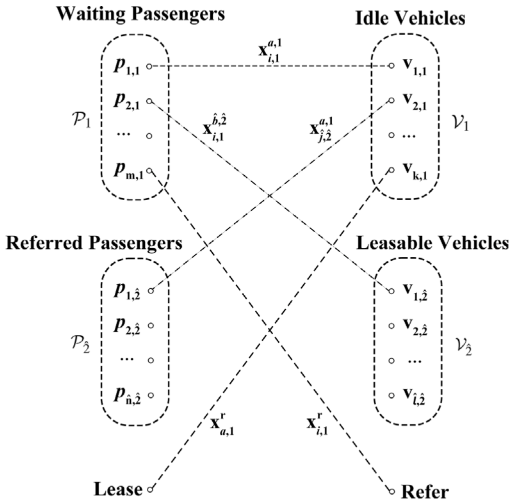

Fig. 1. Visualization of the bipartite graph for the matching optimization problem integrating the referral cooperation mechanism. The graph is from the perspective of platform 1, where the referred passengers and leasable vehicles are offered by platform 2.

of the referred passengers will be matched to at most one vehicle. Eqs. (14) and (15) are similar constraints for the vehicles. Finally, Eq. (16) is the binary constraint for the decision variables.

After solving the optimization, the binary decision variables $x^{a,1}_{i,1}$, $x^{a,1}_{\hat{j},\hat{2}}$, and $x^{\hat{b},\hat{2}}_{i,1}$ indicate the matches between platform 1's own idle vehicles and its own passengers, platform 1's own idle vehicles and referred passengers from platform 2, and platform 1's own passengers and leasable vehicles from platform 2, respectively. The matched passengers are informed of their trip details and they shall make their decisions to accept or decline (based on the Eqs. (1)–(3)). The designated vehicles will then be dispatched to service those accepted passengers.

Consider the referred passengers from platform 2 in set $\mathcal{P}_{\hat{2}}$, for those who are not matched by platform 1, they will be returned to platform 2 and rejoin the set $\mathcal{P}_2$. Similarly, vehicles offered for lease in $\mathcal{V}_{\hat{2}}$ either get successfully leased and become en-route to pick up designated passengers, or they are returned to the platform 2 and rejoin the set $\mathcal{V}_2$. Thus, both $\mathcal{P}_{\hat{2}}$ and $\mathcal{V}_{\hat{2}}$ become empty sets after platform 1's matching.

The binary decision variables $x^{\mathrm{r}}_{i,1}$ and $x^{\mathrm{r}}_{a,1}$, indicate which of platform 1's own passengers and vehicles shall be offered for referral and lease, respectively to platform 2. While the decision variables $f^{\mathrm{r}}_{i,1}$ and $c^{\mathrm{r}}_{a,1}$ determine the referral and lease prices associated. These decision variables, $x^{\mathrm{r}}_{i,1}$, $x^{\mathrm{r}}_{a,1}$, $f^{\mathrm{r}}_{i,1}$, and $c^{\mathrm{r}}_{a,1}$, allow the sets $\mathcal{P}_{\hat{1}}$, $\mathcal{F}_{\hat{1}}$, $\mathcal{V}_{\hat{1}}$, and $\mathcal{C}_{\hat{1}}$ to be constructed for platform 2 to perform the same optimization in its following matching instance. In Section 3.2, we introduce the idea of 'holding' a passenger or vehicle. That is, the platform purposefully does not match certain passengers or vehicles in the current instance, and instead 'holds' them for future rounds of matching. In Sections 3.3 and 3.4, we extend this idea to formulate the expressions for the expected profit of offering referrals and leases, $E[\pi^{\mathrm{r}}_{i,1}]$ and $E[\pi^{\mathrm{r}}_{a,1}]$, as functions of $f^{\mathrm{r}}_{i,1}$ and $c^{\mathrm{r}}_{a,1}$.

3.2. Expected 'holding' profit

Consider a standard batch matching algorithm, where waiting passengers are periodically matched to idle vehicles. It is not guaranteed that every single waiting passenger will be matched to a vehicle, for example when there are more waiting passengers than idle vehicles in the batch. However, even when there are sufficient idle vehicles in the batch, it still may not be preferable to match all the waiting passengers (Ramezani and Valadkhani, 2023). The reason being that it may lead to inefficient matching or WGC problem, where vehicles are dispatched to pick up far-away passengers, which would deteriorate the overall efficiency and lead to fewer available vehicles in the future (Ouyang and Yang, 2023). Conversely, it is also true that it may not be preferable to match all the idle vehicles when there is a surplus of waiting passengers. Under the fare and wage structure considered in this study, where the profit earned depends on the deadhead and could be negative, we consider an expected profit approach to systematically determine which passengers or vehicles should not be matched, and instead 'hold' them for future rounds of matching.

Let us define the value of a passenger as the expected profit to be earned from them. It is clear that the value of a passenger remains even if they are not matched in the current instance, as it is still possible to match them in the future. Though the passenger's residual value, that is the expected profit to be earned from them, given they are not matched in the current instance, may diminish given that there will be an increase in the likelihood of cancellation. If the platform considers the long-term cost of cancellation, the residual value may even become negative. However, when it is anticipated that the residual value is greater than the profit of any of the current matches, then the passenger should be 'held' for future rounds of matching instead of being matched now. In other words, it is plausible and rational for the platform to generate a higher expected profit by intentionally not matching a passenger. Such a scenario is likely due to the available idle vehicles being relatively far away from the passengers in the present which reduces profitability. Note that, this approach may not be directly applicable if the wage of the driver is a percentage based on the fare, since the residual value of a passenger will always be lower than the profit generated by matching them now. Though an artificial penalty could be imposed on the deadhead to utilize the method.

The approach requires the residual value of the passengers or the expected profit of 'holding' the passengers to be determined. Consider passenger $p_{i,1}$ (with origin $O_{i,1}$ and destination $D_{i,1}$), we formulate the expected 'holding' profit of him/her, $E^*[\pi^{\mathrm{h}}_{i,1}]$, as follows:

$$E^*[\pi^{\mathrm{h}}_{i,1}] = E[\mathrm{Pr}^{\mathrm{m}}_{i,1}] \cdot (f_{i,1} - E[c_{i,1}]) - (1 - E[\mathrm{Pr}^{\mathrm{m}}_{i,1}]) \cdot c_{\mathrm{c}}, \tag{17}$$

where $E[\mathrm{Pr}^{\mathrm{m}}_{i,1}]$ is the expected probability that the passenger will eventually be matched and serviced. c_{c} is the long-term cost of cancellation. $E[c_{i,1}]$ is the expected cost of servicing the passenger which is dependent on the expected pickup time of the passenger:

$$E[c_{i,1}] = \lambda_3 E[w_{i,1}] + \lambda_4 |O_{i,1} D_{i,1}|. \tag{18}$$

We estimate the two expected values, $E[\mathrm{Pr}^{\mathrm{m}}_{i,1}]$ and $E[w_{i,1}]$. A platform that has more information on the passengers may obtain a better estimation of $E[\mathrm{Pr}^{\mathrm{m}}_{i,1}]$. For example, if a platform keeps track of a passenger's user history, it may be able to improve the estimation of their patience level, their potential decisions on possible matches, etc. Consequently, the platform can better estimate the probability that the passenger will eventually be matched and serviced, considering Type I and Type II cancellations. In this study, $E[\mathrm{Pr}^{\mathrm{m}}_{i,1}]$ is estimated as the 10-minute moving average number of passenger acceptance divided by the 10-minute moving average number of passenger demand. It implies that the platform does not hold any information of passengers on an individual level, and $E[\mathrm{Pr}^{\mathrm{m}}_{i,1}]$ is only dependent on the immediate past performance of the platform; consequently all passengers in the same batch shall have the same value of $E[\mathrm{Pr}^{\mathrm{m}}_{i,1}]$. There are also many ways of estimating $E[w_{i,1}]$. For example, $E[w_{i,1}]$ can be dependent on each passenger's location; if they are in an area with a higher density of vehicles, they may have a lower expected pickup time. In this study, for the sake of computation efficiency, $E[w_{i,1}]$ is also estimated as the 10-minute moving average of measured pickup times.

Similarly, not matching a vehicle at the current matching instance still generates an expected profit. When formulating the expected profit for 'holding' a vehicle, there are a few points of consideration. Firstly, the income for the platform does not come directly from the vehicles but from passengers, therefore it is only when a vehicle is utilized to service a passenger, the income is realized. Additionally, we do not need to worry about vehicles leaving the platform, such as passenger cancellations, as it is assumed that the vehicles will always remain in the market. We consider a vehicle, $v_{a,1}$, is matched and dispatched now, then it will be utilized from now, t_0, to $t_0 + \Delta t_1$, to service passenger, $p_{i,1}$, and generate a profit of $\pi^{a,1}_{i,1}$. However, if the same vehicle is dispatched to the same passenger generating the same profit but at a later time $t_0 + \Delta t_2$ ($\Delta t_2 < \Delta t_1$), then the vehicle is not fully utilized in the time period t_0 to $t_0 + \Delta t_1$. Hence we assume that the equivalent expected profit of matching the vehicle in the future is $\frac{\Delta t_1 - \Delta t_2}{\Delta t_1} \pi^{a,1}_{i,1}$. We thus formulate the expected profit of 'holding' the vehicle, $E^*[\pi^{\mathrm{h}}_{a,1}]$, as follows:

$$E^*[\pi^{\mathrm{h}}_{a,1}] = E[\pi_{a,1}] E[\mathrm{Pr}^{\mathrm{m}}_{a,1}] \sum_{s=1}^{T} \frac{T-s}{T} (1 - E[\mathrm{Pr}^{\mathrm{m}}_{a,1}])^{s-1}, \tag{19}$$

where $E[\pi_{a,1}]$ is the profit that the vehicle is expected to earn for the platform, $E[\mathrm{Pr}^{\mathrm{m}}_{a,1}]$ is the expected probability that the vehicle will be matched in each of the following matching instances, and T is the number of matching instances in the average time a vehicle is utilized to service one passenger. We estimate $E[\pi_{a,1}]$ by measuring the 10-minute moving average of profits earned by the vehicles; while $E[\mathrm{Pr}^{\mathrm{m}}_{a,1}]$ is estimated as the probability a vehicle is matched in the previous matching instance.

Finally, as the expected 'holding' profits for passengers and vehicles are established, it is important to realize that each trip is comprised of one passenger and one vehicle, and the matching algorithm needs to factor it in. As an example, consider a passenger and a vehicle each have an expected 'holding' profit of \$1, and the platform can also generate \$1 profit by matching them. Then the choices of 'holding' them or matching them should be indifferent to the platform. If the passenger and the vehicle have different expected 'holding' profits, then we assume the indifferent matching profit is the average of the two values. Therefore, the expected 'holding' profits of the passenger and the vehicle perceived by the platform, $E[\pi^{\mathrm{h}}_{i,1}]$ and $E[\pi^{\mathrm{h}}_{a,1}]$, are

$$E[\pi^{\mathrm{h}}_{i,1}] = 0.5 E^*[\pi^{\mathrm{h}}_{i,1}] \tag{20}$$

$$E[\pi^{\mathrm{h}}_{a,1}] = 0.5 E^*[\pi^{\mathrm{h}}_{a,1}]. \tag{21}$$

3.3. Expected passenger referral profit

Logically, if a platform is expected to generate a higher referral profit than any of the possible matches to a passenger or to 'hold' the passenger, then there is no reason to service this passenger itself. This is reflected in the optimization in Eqs. (11)–(16). Therefore, in order for the platform to decide whether to refer a passenger, as well as the associated referral prices, it is necessary to determine the expected profits to be generated for those actions. From the perspective of platform 1 as the referrer, we formulate the expected profit for referring passenger $p_{i,1}$ to platform 2, $E[\pi^{\mathrm{r}}_{i,1}]$, as follows:

$$E[\pi^{\mathrm{r}}_{i,1}] = E[\mathrm{Pr}^{\mathrm{r}}_{i,1}] \cdot \pi^{\mathrm{r}}_{i,1} + (1 - E[\mathrm{Pr}^{\mathrm{r}}_{i,1}]) \cdot E[\pi^{\mathrm{h}}_{i,1}]. \tag{22}$$

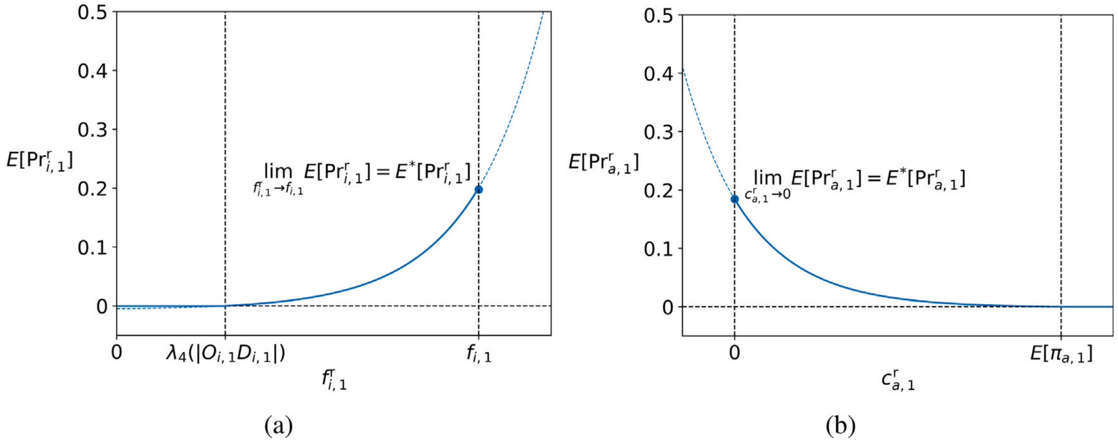

Fig. 2. (a) The expected probability of the other platform accepting the passenger referral ($E[\mathrm{Pr}^{\mathrm{r}}_{i,1}]$) as a function of the referral price ($f^{\mathrm{r}}_{i,1}$). $E^*[\mathrm{Pr}^{\mathrm{r}}_{i,1}]$ is the highest possible expected probability the referral would be accepted. (b) The expected probability of the other platform accepting the vehicle lease ($E[\mathrm{Pr}^{\mathrm{r}}_{a,1}]$) as a function of the lease price ($c^{\mathrm{r}}_{a,1}$). $E^*[\mathrm{Pr}^{\mathrm{r}}_{a,1}]$ is the highest possible expected probability the lease would be accepted.

In Eq. (22), $E[\mathrm{Pr}^{\mathrm{r}}_{i,1}]$ is the expected probability of platform 2 accepting the referral. Given the acceptance, $\pi^{\mathrm{r}}_{i,1}$ is the referral profit to be generated where $\pi^{\mathrm{r}}_{i,1} = f_{i,1} - f^{\mathrm{r}}_{i,1}$. If the referral is not accepted, then passenger $p_{i,1}$ is reconsidered in platform 1, and the passenger will still be possible to be matched. Hence, the residual value of the passenger remains even if the referral is not successful. This residual value is equivalent to the perceived expected profit of 'holding' this passenger now to match them in the future, $E[\pi^{\mathrm{h}}_{i,1}]$.

Since $f^{\mathrm{r}}_{i,1}$ is a decision variable, without any optimization, it is straightforward to show that if $f^{\mathrm{r}}_{i,1}$ is selected such that $\pi^{\mathrm{r}}_{i,1} = E[\pi^{\mathrm{h}}_{i,1}]$, then $E[\pi^{\mathrm{r}}_{i,1}] = E[\pi^{\mathrm{h}}_{i,1}]$. Therefore, we show that optimizing $f^{\mathrm{r}}_{i,1}$ will generate an expected referral profit at least equal to the perceived expected profit of 'holding' the passenger. Ergo, the option to refer is always as good as the option to 'hold' a passenger if not superior. Consequently, the optimization problem in Eqs. (11)–(16) does not consider the option to 'hold'. It therefore implies, when comparing to a non-cooperating duopoly, whenever a passenger has to be 'hold' for future matching, whether due to it being non-profitable, or there are simply not enough vehicles, the cooperation mechanism that allows the option to refer this passenger can always be expected to generate an equal or higher profit.

To complete the formulation of Eq. (22), we need to obtain the expected probability of platform 2 accepting the referral, $E[\mathrm{Pr}^{\mathrm{r}}_{i,1}]$, which is a function of $f^{\mathrm{r}}_{i,1}$. The lower the $f^{\mathrm{r}}_{i,1}$, which is the fee paid to platform 2 to service the passenger, the less likely platform 2 would accept the referral and vice versa. The two critical values of $f^{\mathrm{r}}_{i,1}$ are $f^{\mathrm{r}}_{i,1} = \lambda_4|O_{i,1}D_{i,1}|$ and $f^{\mathrm{r}}_{i,1} = f_{i,1}$. For $f^{\mathrm{r}}_{i,1} \le \lambda_4|O_{i,1}D_{i,1}|$, $E[\mathrm{Pr}^{\mathrm{r}}_{i,1}] = 0$, since $\lambda_4|O_{i,1}D_{i,1}|$ is the minimum possible wage that has to be paid to the driver. For $f^{\mathrm{r}}_{i,1} \ge f_{i,1}$, platform 1 would make a non-positive profit if the referral is accepted, hence there is no reason for platform 1 to offer a fee higher than the amount charged to the passenger. We assume that between the range $\lambda_4|O_{i,1}D_{i,1}| \le f^{\mathrm{r}}_{i,1} \le f_{i,1}$, $E[\mathrm{Pr}^{\mathrm{r}}_{i,1}]$ follows a transformed exponential function:

$$E[\mathrm{Pr}^{\mathrm{r}}_{i,1}] = \frac{(e^{f^{\mathrm{r}}_{i,1} - \beta_1}) - 1}{\beta_2}, \tag{23}$$

where $\beta_1 = \lambda_4|O_{i,1}D_{i,1}|$ and β_2 depends on the limiting value of $E[\mathrm{Pr}^{\mathrm{r}}_{i,1}]$ (see Fig. 2(a)). We denote the limiting value of $E[\mathrm{Pr}^{\mathrm{r}}_{i,1}]$ as $E^*[\mathrm{Pr}^{\mathrm{r}}_{i,1}]$, which is the highest possible expected probability that platform 2 will accept the referral. It occurs when platform 1 offers to pay the full fare to platform 2 and is not making a referral profit. When platform 1 offers to pay the full fare, the referred passenger is indifferent to platform 2's own passengers from platform 2's perspective. Therefore, we assume $E^*[\mathrm{Pr}^{\mathrm{r}}_{i,1}]$ is estimated using 10-minute average matchings divided by the average number of waiting passengers. However, the $f^{\mathrm{r}}_{i,1}$ that maximizes Eq. (22) is independent of β_2, as we take the derivative $\frac{\mathrm{d}(E[\pi^{\mathrm{r}}_{i,1}])}{\mathrm{d}(f^{\mathrm{r}}_{i,1})} = 0$, which yields:

$$f^{\mathrm{r}}_{i,1} = W(e^{\beta_1 - f_{i,1} + E[\pi^{\mathrm{h}}_{i,1}] + 1}) + f_{i,1} - E[\pi^{\mathrm{h}}_{i,1}] - 1, \tag{24}$$

where $W(.)$ is the product log function. Therefore, by solving Eq. (24), the optimal price of referral, $f^{\mathrm{r}*}_{i,1}$, can be determined. Consequently, by solving Eq. (22) at such price, the maximum expected profit of referral, $E^*[\pi^{\mathrm{r}}_{i,1}]$, can be obtained.

3.4. Expected vehicle lease profit

Similarly, when the platform offers to lease vehicle $v_{a,1}$, an expected profit, $E[\pi^{\mathrm{r}}_{a,1}]$ is to be generated, which is formulated as follows:

$$E[\pi^{\mathrm{r}}_{a,1}] = E[\mathrm{Pr}^{\mathrm{r}}_{a,1}] \cdot \pi^{\mathrm{r}}_{a,1} + (1 - E[\mathrm{Pr}^{\mathrm{r}}_{a,1}]) \cdot E[\pi^{\mathrm{h}}_{a,1}]. \tag{25}$$

Table 1
Dynamic model passenger choice parameters. μ, σ, a, and b are the mean, standard deviation, lower bound, and upper bound, respectively for the bounded normal distributions.

Parameter	Description	Unit	μ	σ	a	b
$\bar{m}$	Passenger matching patience	s	60	10	40	80
u^{o}	Utility constant for other travel modes	–	−6	0.5	−7	−5
β^{s}	Utility constant for ridesourcing trip	–	0	0	–	–
β^{w}	Utility coefficient for pickup time	1/min	1	0	–	–
β^{f}	Utility coefficient for trip fare	1/$	3.2	0.2	2.8	3.6
λ_1^{f}	Fair fixed base price	$	2.55	0	–	–
λ_4^{f}	Fair trip duration variable price	$/min	0.6	0	–	–

$E[\mathrm{Pr}^{r}_{a,1}]$ is the probability of platform 2 accepting the lease offer, which is dependent on the lease price $c^{r}_{a,1}$. Recall that $c^{r}_{a,1} = \pi^{r}_{a,1}$. Similar to the previous section, it is straightforward to show that an optimized $c^{r}_{a,1}$ will generate an expected lease profit at least equal to the perceived expected profit of 'holding' the vehicle. Additionally, the higher the $c^{r}_{a,1}$, the less likely platform 2 would accept the offer. We assume that if $c^{r}_{a,1}$ exceeds the profit utilizing a vehicle is expected to earn, $E[\pi_{a,1}]$, then platform 1 will expect platform 2 to not accept the lease. Furthermore, $c^{r}_{a,1}$ should be non-negative for platform 1 to generate a lease profit. Hence, for $c^{r}_{a,1} > E[\pi_{a,1}]$, $E[\mathrm{Pr}^{r}_{a,1}] = 0$, and for $0 \leq c^{r}_{a,1} \leq E[\pi_{a,1}]$, we assume $E[\mathrm{Pr}^{r}_{a,1}]$ to have the following functional form as is visualized in Fig. 2(b):

$$E[\mathrm{Pr}^{r}_{a,1}] = \frac{(e^{\beta_3 - c^{r}_{a,1}}) - 1}{\beta_4}, \tag{26}$$

where $\beta_3 = E[\pi_{a,1}]$, and β_4 is dependent on the calibrated highest expected probability that platform 2 will accept a vehicle lease, $E^{*}[\mathrm{Pr}^{r}_{a,1}]$, which occurs when there is no associated cost. Again, when platform 1 offers to lease a vehicle with no associated cost, this vehicle is indifferent to platform 2's own vehicles from platform 2's perspective. Therefore, we assume $E^{*}[\mathrm{Pr}^{r}_{a,1}]$ is estimated using 10-minute average matchings divided by the average number of idle vehicles. The $c^{r}_{a,1}$ that maximizes Eq. (25) is independent of β_4, as we take the derivative $\frac{\mathrm{d}(E[\pi^{r}_{a,1}])}{\mathrm{d}(c^{r}_{a,1})} = 0$, which yields:

$$c^{r}_{a,1} = -W(e^{-\beta_3 + E[\pi^{h}_{a,1}]+1}) + E[\pi^{h}_{a,1}] + 1. \tag{27}$$

We denote this optimal lease price as $c^{r*}_{a,1}$. Subsequently, solving Eq. (25) at such lease price allows the highest expected vehicle lease profit, $E^{*}[\pi^{r}_{a,1}]$ to be determined.

3.5. Transformation of the optimization problem

For each individual waiting passenger, we can obtain the optimal values for the passenger referral price and the corresponding maximized expected passenger referral profit, i.e., $f^{r*}_{i,1}$ and $E^{*}[\pi^{r}_{i,1}]$, respectively. For each individual idle vehicle, we can obtain the vehicle lease price and the corresponding maximized expected vehicle lease profit, i.e., $c^{r*}_{a,1}$ and $E^{*}[\pi^{r}_{a,1}]$, respectively.

Consequently, we can transform Eq. (11) into an equivalent optimization problem as follows, with the same constraints as in Eqs. (12)–(16), and with $f^{r}_{i,1} = f^{r*}_{i,1}\ \ \forall p_{i,1} \in \mathcal{P}_1$ and $c^{r}_{a,1} = c^{r*}_{a,1}\ \ \forall v_{a,1} \in \mathcal{V}_1$:

$$\begin{aligned} \max_{x^{a,1}_{i,1}, x^{a,1}_{\hat{j},\hat{2}}, x^{\hat{b},\hat{2}}_{i,1}, x^{r}_{i,1}, x^{r}_{a,1}} \quad & \sum_{\mathcal{E}_1} \pi^{a,1}_{i,1} x^{a,1}_{i,1} + \sum_{\bar{\mathcal{E}}_1} \pi^{a,1}_{\hat{j},\hat{2}} x^{a,1}_{\hat{j},\hat{2}} + \sum_{\tilde{\mathcal{E}}_1} \pi^{\hat{b},\hat{2}}_{i,1} x^{\hat{b},\hat{2}}_{i,1} + \sum_{i=1}^{m} E^{*}[\pi^{r}_{i,1}] x^{r}_{i,1} + \sum_{a=1}^{k} E^{*}[\pi^{r}_{a,1}] x^{r}_{a,1} \\ \text{s.t.} \quad & (12)\text{-}(16) \end{aligned} \tag{28}$$

4. Numerical experiments

4.1. Experiment setup

We test the proposed cooperation mechanism in a simulator, where the Manhattan New York road network is transformed into a directed graph. We assume that every link in the road network has a unique and time-invariant speed at which vehicles travel, i.e. the effects of congestion (Alisoltani et al., 2021; Beojone and Geroliminis, 2021, 2023a; Zhang and Zhang, 2022; Zhang and Nie, 2022) are not considered in this study. For the passenger choice model introduced in Section 2.1, they are consistent for all test cases considered. Some of the parameter values for each individual passenger are drawn from bounded normal distributions. The mean, standard deviations, and bounds for those parameters are shown in Table 1.

We consider 7 test cases, the platforms adopt the same pricing and wage parameters for the first 6 test cases. In the final test case, we consider that one of the platforms' prices is slightly higher than the other one. The parameter values associated with pricing and wage structures (Section 2.3) in all 7 test cases are shown in Table 2.

We consider four symmetrical duopoly test cases, where the fleet sizes operated by both platforms are identical, and the probability of the passengers choosing each platform is 50%. We also consider three asymmetrical duopoly test cases, where the

Table 2
Platform pricing and wage structure parameters for the 7 test cases.

Platform	Parameter	Description	Unit	Test cases 1–6	Test case 7
1	$\lambda_{1,1}$	Fixed base price	\$	2.55	2.55
	$\lambda_{2,1}$	Trip duration variable price	\$/min	0.6	0.6
	$\lambda_{3,1}$	Wage per unit time driver spent matched	\$/min	0.48	0.48
	$\lambda_{4,1}$	Wage per unit time driver spent occupied	\$/min	0.48	0.48
2	$\lambda_{1,2}$	Fixed base price	\$	2.55	3
	$\lambda_{2,2}$	Trip duration variable price	\$/min	0.6	0.6
	$\lambda_{3,2}$	Wage per unit time driver spent matched	\$/min	0.48	0.48
	$\lambda_{4,2}$	Wage per unit time driver spent occupied	\$/min	0.48	0.48

Table 3
The supply and demand for the six test cases considered. We denote the fleet sizes for each of the two platforms as V_1^t and V_2^t respectively. We denote the probability of passengers choosing each of the two platforms as Pr_1 and Pr_2, respectively.

Test case no.	V_1^t	V_2^t	Pr_1	Pr_2
1	1600	1600	50%	50%
2	2000	2000	50%	50%
3	2400	2400	50%	50%
4	2400	1600	60%	40%
5	2400	1600	50%	50%
6	2400	1600	40%	60%
7	2000	2000	50%	50%

fleet sizes for the two platforms and/or the probability of the passengers choosing each platform are different. We summarize the details of the test cases in Table 3. We use real passenger demand data from 7–11 am on 3/Feb/2015 (Tuesday) based on New York City yellow taxi trip records (71692 trip requests). The data includes the time, origin, and destination of each passenger request. We assume the passengers select one of the platforms based on given probabilities. Vehicles are added at random locations at the beginning of the simulation until the desired fleet size is achieved. The total rate of vehicle addition into the market is 4 vehicles per second. When they are added to the network, they start to randomly cruise until they are matched and dispatched to pick up a passenger. Once the passenger is dropped off, the vehicle becomes idle again, and we assume they become stationary at the drop off location. From the start of each simulation, we allow for 40 min of warm-up period, any performance indicators are taken after this period (roughly 62000 trip requests in the remaining time). The results are obtained by averaging 5 simulations.

For each test case, we compare the proposed mechanism to two benchmark scenarios which are an equivalent non-cooperating (standard) duopoly and an equivalent monopoly (identical total demand and supply). The equivalent non-cooperating (standard) duopoly has the same setup as shown in Table 3. Whereas the equivalent monopoly operates a fleet with a size equal to the sum of the two platforms in the duopoly while servicing the total demand. To conduct fair comparisons, we assume that the passenger choice parameters, as well as the platform pricing and wage parameters, are the same for all scenarios and are as already set out in Tables 1 and 2. The platforms also adopt the same matching interval, Δ, at 10 s. Additionally, the platforms all consider the same long-term cost of passenger cancellation, c_c, which is assumed to be \$4.6 (Jiao and Ramezani, 2022). Also for conducting fair comparisons, we assume for the benchmarks, the platforms all adopt a matching algorithm that maximizes the expected profits with the consideration of 'holding' passengers or vehicles for future rounds of matching. This can be formulated as follows:

$$\max_{x_{i,1}^{a,1}, x_{i,1}^{h}, x_{a,1}^{h}} \sum_{\mathcal{E}_1} \pi_{i,1}^{a,1} x_{i,1}^{a,1} + \sum_{i=1}^{m} E[\pi_{i,1}^{h}] x_{i,1}^{h} + \sum_{a=1}^{k} E[\pi_{a,1}^{h}] x_{a,1}^{h} \tag{29}$$

$$\text{s.t.} \quad \sum_{a=1}^{k} x_{i,1}^{a,1} + x_{i,1}^{h} = 1 \quad \forall\, p_{i,1} \in \mathcal{P}_1 \tag{30}$$

$$\sum_{i=1}^{m} x_{i,1}^{a,1} + x_{a,1}^{h} = 1 \quad \forall\, v_{a,1} \in \mathcal{V}_1 \tag{31}$$

$$x_{i,1}^{a,1}, x_{i,1}^{h}, x_{a,1}^{h} \in \{0,1\} \tag{32}$$

where $x_{i,1}^{h}$ and $x_{a,1}^{h}$ are the decision variables to 'hold' passenger $p_{i,1}$ and vehicle $v_{a,1}$, respectively.

4.2. Symmetrical duopolies

4.2.1. Performance of the cooperation mechanism

For each of the test cases (1) - (3), we compare the performance indicators of the cooperation mechanism to the standard (non-cooperative) duopoly and monopoly scenarios. The results are shown in Tables 4, 5, and 6. Note we repeat 5 simulation experiments for each scenario to obtain average performance indicators.

Table 4
Performance of the cooperation mechanism in comparison with standard symmetrical duopoly and monopoly for the test case (1): $V_1^t = V_2^t = 1600, \mathrm{Pr}_1 = \mathrm{Pr}_2 = 50\%$.

Performance indicators	Duopoly	Cooperation mechanism	Monopoly
Average passenger matching time[a] (s)	14.4	15.5	15.1
Average passenger pickup time[b] (s)	202.9	186.6 (−8.0%)	175.5 (−13.5%)
Average passenger in-vehicle time (s)	830.8	837.4	860.4
Type I Cancellations (impatience)	20 469	20 968	22 578
Type II Cancellations (dissatisfactory service)	6150	5103	4275
Total passenger cancellations	26 619	26 071 (−2.1%)	26 853 (0.9%)
Number of serviced passengers	35 735	36 277	35 447
Total platform profit ($)	97 763	103 772 (6.1%)	105 436 (7.8%)
Potential future loss due to cancellations ($)	−122448	−119926	−123520
Average vehicle idle time	6.2%	5.3%	6.1%
Average vehicle dispatched time	16.5%	15.5%	14.3%
Average vehicle occupied time	77.3%	79.2%	79.6%
Drivers' average wage ($/hr)	27.01	27.26	27.04

[a] Average passenger matching time is defined as the average time between a passenger entering the platform until they are matched by the platform, excluding those who canceled before being matched (Type 1 Cancellation). Note that due to this definition, there is not a direct positive relationship between type 1 cancellation, and the average matching time.

[b] Average passenger pickup time is defined as the average time between the passenger being matched to the indicative time that the vehicle will arrive, including those who are not satisfied with the trip (Type 2 Cancellation).

Table 5
Performance of the cooperation mechanism in comparison with standard symmetrical duopoly and monopoly for the test case (2): $V_1^t = V_2^t = 2000, \mathrm{Pr}_1 = \mathrm{Pr}_2 = 50\%$.

Performance indicators	Duopoly	Cooperation mechanism	Monopoly
Average passenger matching time (s)	11.3	12.0	12.2
Average passenger pickup time (s)	190.5	180.2 (−5.4%)	166.5 (−12.6%)
Average passenger in-vehicle time (s)	796.1	799.6	812.0
Type I Cancellations (impatience)	13 609	13 622	15 297
Type II Cancellations (dissatisfactory service)	5968	5262	4092
Total passenger cancellations	19 577	18 884 (−3.5%)	19 389 (−1%)
Number of serviced passengers	42 750	43 642	42 921
Total platform profit ($)	116 884	122 015 (4.4%)	125 180 (7.1%)
Potential future loss due to cancellations ($)	−90054	−86866	−89189
Average vehicle idle time	13.6%	12.6%	13.5%
Average vehicle dispatched time	15.2%	14.7%	13.5%
Average vehicle occupied time	71.2%	72.7%	73.0%
Drivers' average wage ($/hr)	24.88	25.17	24.91

Table 6
Performance of the cooperation mechanism in comparison with standard symmetrical duopoly and monopoly for the test case (3): $V_1^t = V_2^t = 2400, \mathrm{Pr}_1 = \mathrm{Pr}_2 = 50\%$.

Performance indicators	Duopoly	Cooperation mechanism	Monopoly
Average passenger matching time (s)	9.8	10.4	10.8
Average passenger pickup time (s)	179.2	171.2 (−4.5%)	160.5 (−10.4%)
Average passenger in-vehicle time (s)	787.4	789.8	797.8
Type I Cancellations (impatience)	10 443	10 555	11 879
Type II Cancellations (dissatisfactory service)	5497	4928	4138
Total passenger cancellations	15 940	15 482 (−2.9%)	16 017 (0.5%)
Number of serviced passengers	46 467	46 884	46 340
Total platform profit ($)	129 331	133 366 (3.1%)	135 810 (5.0%)
Potential future loss due to cancellations ($)	−73324	−71219	−73678
Average vehicle idle time	23.1%	22.7%	23.7%
Average vehicle dispatched time	13.0%	12.6%	11.7%
Average vehicle occupied time	63.9%	64.7%	64.6%
Drivers' average wage ($/hr)	22.14	22.25	21.97

Table 7
Passenger referral and vehicle lease statistics for the test cases (1)–(3).

	Test case (1)	Test case (2)	Test case (3)
Total successful passenger referrals	5839	2877	2444
Average passenger referral profit ($)	0.55	1.11	1.35
Total passenger referral profit ($)	3213	3187	3295
Total successful vehicle leases	2277	3965	4464
Average vehicle leases profit ($)	1.87	1.76	1.62
Total vehicle leases profit ($)	4267	6959	7246
Total profit ($)	103 772	122 015	133 366
Total profit generated from cooperation ($)	7480	10 146	10 541
Proportion of cooperation profit relative to total profit	7.2%	8.3%	7.9%

It can be observed that inefficiencies due to market fragmentation do exist. Using test case (2) as an example (Table 5), comparing the monopoly to the duopoly, the average passenger pickup time is reduced by 24 s (12.6%), the number of cancellations is decreased by 188 (1%), and an additional $8296 (7.1%) of total profit is generated. Though the monopoly does not outperform the duopoly substantially, it does act as a benchmark for the upper limit of the cooperation mechanism.

The proposed cooperation mechanism between the two platforms in the duopoly improves all major performance indicators compared to those without cooperation for all three test cases. Again, using test case (2) as the example, the mechanism reduces average passenger pickup time by 10.3 s (5.4%) compared to the standard duopoly, while being 13.7 s (8.2%) more than the monopoly. We see a $5131 (4.4%) increase in profit, which is only $3165 (2.5%) short of the monopoly. Surprisingly, we observe the number of cancellations from the cooperating duopoly is even lower than that in the monopoly (which also occurs in the other two test cases). There is a 693 (3.5%) and a 505 (2.6%) reduction in cancellation compared to the standard duopoly and the monopoly, respectively. From the driver's perspective under the cooperation mechanism, we can observe that they spent marginally more time with passengers onboard, and less time deadheading or being idle compared to the standard duopoly. Consequently, the hourly wage of drivers is increased by $0.29 (1.2%). This hourly wage is higher than that observed in the monopoly for all three test cases. This is due to the fact that drivers are assumed to be paid for deadheading.

There is a distinct correlation between the number of cancellations due to dis-satisfactory service and average passenger pickup time. It is logical that as the average passenger pickup times decrease in the cooperating duopoly and further in the monopoly, the quality of service improves and the number of cancellations due to dis-satisfactory service reduces. At the same time, there is also a subtle correlation between the number of cancellations due to impatience and the average passenger in-vehicle time. As passenger in-vehicle time increases, the vehicles spend more time en-route which would lead to a reduced number of idle vehicles and thus reduced matching rate. Consequently, the number of cancellations due to impatience would increase. This is observed for all three test cases. Therefore, as the cooperation mechanism reduces the average passenger pickup time, while maintaining roughly the same average passenger in-vehicle time, it yields the lowest number of total cancellations. We argue that the apparent increase in the average passenger in-vehicle time in the monopoly is a consequence of the profit maximizing algorithm. When there are more matching options for the monopoly, the matching algorithm tends to match passengers with longer trips which would generate more revenue. It implies that the monopoly is implicitly favoring passengers with longer trips while foregoing the potential of servicing more passengers. This is also evident that as the total number of vehicles decreases, the average passenger in-vehicle time increases, as when supply is scarce, the platforms favor passengers with longer trips.

4.2.2. Properties of the cooperation mechanism

For test cases (1) - (3), when the cooperation mechanism is in place, we document the number of total successful passenger referrals (vehicle leases) conducted by the two platforms together, the average passenger referral (vehicle lease) profit, and the total passenger referral (vehicle lease) profit. Additionally, we also document the total profit generated by the cooperation mechanism from the two platforms, and the proportion of cooperation profit relative to total profit. These statistics are shown in Table 7.

Fig. 3 shows one platform's successful passenger referrals and vehicle leases, and the average profit per instance, over time, for each of the three test cases. Note that the referral & lease profits shown for the platform, are implied by the prices set by the platform, which is determined through the optimization problem in Eqs. (11)–(16). At the same time, they are the additional costs incurred by the other platform as it accepts the referral & lease offers. Furthermore, since the two platforms are symmetrical and adopt the same cooperation mechanism, the results from the other platform are similar to the ones shown due to symmetry, and therefore are omitted.

It is interesting to point out a few observations. First, as the market goes from under-supplied (test case (1)) to over-supplied (test case (3)), the platforms' passenger referral profit (price) increases, while the vehicle lease profit (price) decreases. This is intuitive, as vehicles are more valuable when the market is under-supplied, while passengers are more valuable when the market is over-supplied. Secondly, again as the market goes from under-supplied to over-supplied, the platforms' successful passenger referrals decrease, while successful vehicle leases increase. This is primarily due to the profit (price) set by the mechanism, as a higher profit (price) discourages referral/lease acceptance. Lastly, the average vehicle lease profit (price) is generally higher than the average passenger referral profit (price). This is due to the consideration of the long-term cost of passenger cancellation in the optimization problem, which reduces the expected residual value of passengers. Consequently, the platforms are willing to make a lower referral profit to get the passengers serviced.

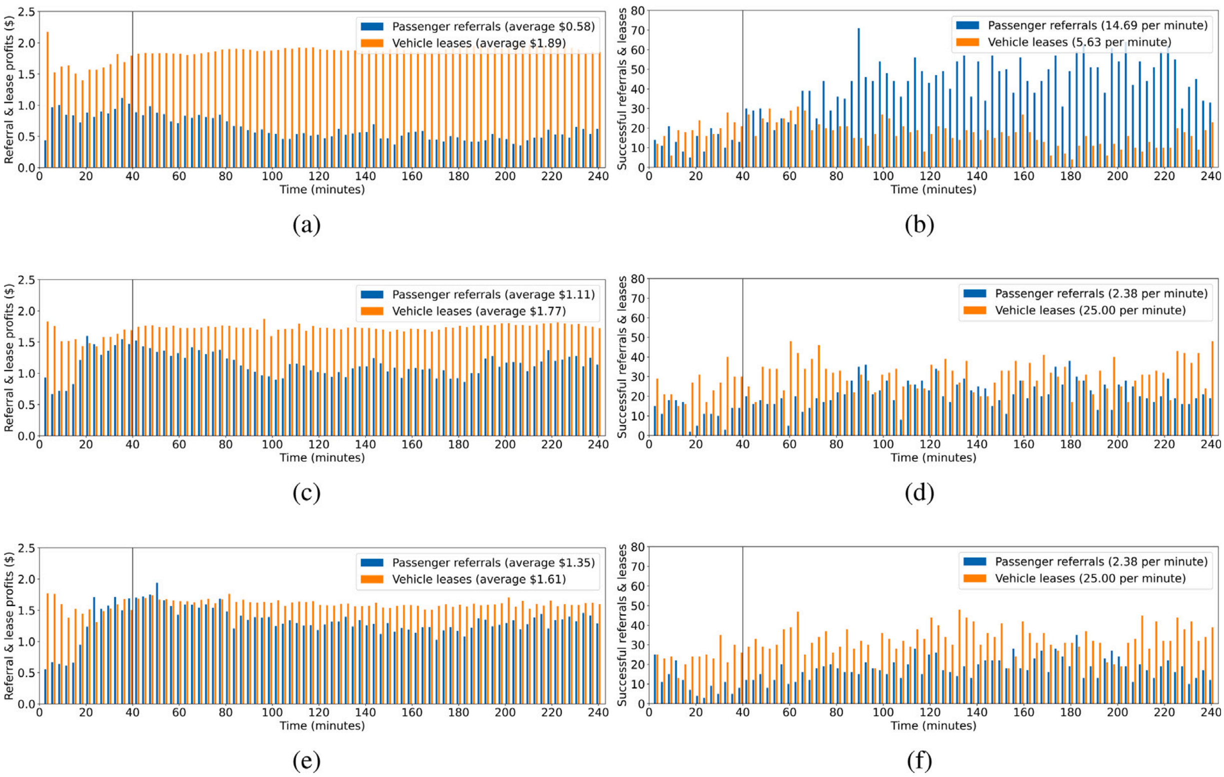

Fig. 3. One platform's successful passenger referrals and vehicle leases, and the average profit per instance, over time, for each of the three test cases. The values are determined in 3-minute intervals. (a) and (b): Test case (1); (c) and (d): Test case (2); and (e) and (f): Test case (3).

Table 8
Performance of the cooperation mechanism in comparison with standard asymmetrical duopoly for the test case (4): $V_1^t = 2400, V_2^t = 1600, \mathrm{Pr}_1 = 60\%, \mathrm{Pr}_2 = 40\%$.

Performance indicators	Standard Duopoly			Cooperation mechanism		
	Platform 1	Platform 2	Combined[a]	Platform 1	Platform 2	Combined[a]
Average passenger matching time (s)	11.8	10.6	11.2	11.9	12.0	12.0
Average passenger pickup time (s)	183.6	199.7	191.7	176.5	185.0	180.8 (−5.7%)
Average passenger in-vehicle time (s)	802.0	793	797.5	802.1	799.3	800.7
Type I Cancellations	8634	5203	13 837	8322	5551	13 873
Type II Cancellations	3211	2627	5838	2904	2166	5070
Total passenger cancellations	11 845	7830	19 765	11 226	7717	18 943 (−4.2%)
Number of serviced passengers	25 763	16 967	42 730	26 056	17 395	43 451
Total platform profit ($)	71 668	45 106	116 774	73 754	48 201	121 956 (4.4%)
Average vehicle idle time	13.3%	14.0%	13.6%	13.1%	11.9%	12.6%
Average vehicle dispatched time	14.8%	15.8%	15.2%	14.3%	15.1%	14.6%
Average vehicle occupied time	72.0%	70.2%	71.3%	72.6%	73.0%	72.8%
Drivers' average wage ($/hr)	24.98	24.76	24.89	25.04	25.34	25.17

[a] Combined means the sum or average of the two platforms depending on the indicator.

4.3. Asymmetrical duopolies

4.3.1. Performance of the cooperation mechanism

Similar to previously, for each of the test cases (4) - (6), we compare the performance indicators of the cooperation mechanism to the standard (non-cooperative) duopoly and monopoly scenarios (note that the equivalent monopoly for cases (4) - (6) is the same as for case (2) shown in Table 5). The results are shown in Tables 8, 9, and 10.

It can be observed that as the asymmetry increases, i.e., from test case (4) to test case (6), the overall performances of the standard duopolies worsen. This is intuitive, as the increased asymmetry leads to more supply and demand imbalance for each of the platforms in the duopolies. Notably, we observe that the implementation of the cooperation mechanism improves the overall performance of the duopoly to roughly the same level regardless of the level of asymmetry. In other words, the cooperation mechanism is capable of

Table 9
Performance of the cooperation mechanism in comparison with standard asymmetrical duopoly for the test case (5): $V_1^t = 2400, V_2^t = 1600, \mathrm{Pr}_1 = \mathrm{Pr}_2 = 50\%$.

Performance indicators	Standard duopoly			Cooperation mechanism		
	Platform 1	Platform 2	Combined	Platform 1	Platform 2	Combined
Average passenger matching time (s)	9.7	14.4	12.08	11.7	12.3	12.0
Average passenger pickup time (s)	182.1	203.9	193.0	178.4	184.0	181.2 (−6.1%)
Average passenger in-vehicle time (s)	784.1	833.4	808.7	792.9	807.3	800.0
Type I Cancellations	5139	10 508	15 647	6369	7304	13 673
Type II Cancellations	2885	3130	6015	2631	2692	5323
Total passenger cancellations	8024	13 638	21 662	9000	9996	18 996 (−12.3%)
Number of serviced passengers	22 956	17 782	40 738	21 899	21 493	43 392
Total platform profit ($)	63 338	48 757	112 095	67 751	53 816	121 567 (8.4%)
Average vehicle idle time	24.1%	6.5%	17.1%	14.7%	10.0%	12.8%
Average vehicle dispatched time	13.0%	16.4%	14.4%	14.3%	15.2%	14.7%
Average vehicle occupied time	62.9%	77.1%	68.6%	71.0%	74.9%	72.5%
Drivers' average wage ($/hr)	21.86	26.93	23.89	24.56	25.95	25.12

Table 10
Performance of the cooperation mechanism in comparison with standard asymmetrical duopoly for the test case (6): $V_1^t = 2400, V_2^t = 1600, \mathrm{Pr}_1 = 40\%, \mathrm{Pr}_2 = 60\%$.

Performance indicators	Standard duopoly			Cooperation mechanism		
	Platform 1	Platform 2	Combined	Platform 1	Platform 2	Combined
Average passenger matching time (s)	8.2	17.5	12.8	11.7	12.3	12.0
Average passenger pickup time (s)	174.2	197.0	185.6	177.5	181.5	179.5 (−3.3%)
Average passenger in-vehicle time (s)	780.6	895.3	838.0	791.4	803.0	797.2
Type I Cancellations	2921	17 426	20 347	4717	8906	13 623
Type II Cancellations	2250	2807	5057	2222	2945	5167
Total passenger cancellations	5171	20 233	25 404	6939	11 851	18 790 (−26.0%)
Number of serviced passengers	19 638	17 279	36 917	18 064	25 531	43 595
Total platform profit ($)	55 261	50 328	105 589	63 476	58 864	122 340 (15.9%)
Average vehicle idle time	35.8%	4.3%	23.2%	14.9%	9.3%	12.6%
Average vehicle dispatched time	10.6%	15.3%	12.5%	14.3%	15.2%	14.6%
Average vehicle occupied time	53.6%	80.3%	64.3%	70.9%	75.5%	72.7%
Drivers' average wage ($/hr)	18.49	27.56	22.12	24.52	26.13	25.17

effectively bridging the supply and demand imbalance between the two platforms. Consequently, we observe that the improvement due to the cooperation mechanism can be substantial. For example, under test case (6), the cooperation mechanism leads to a 26% reduction in passenger cancellations, a 15.9% increase in the total profit generated by the platforms, and a 13.8% increase in drivers' average wage per hour.

Looking at each of the two platforms separately, since platform 1 has a higher fleet size, we can see that it performs better than platform 2 as expected. For all three test cases, after the implementation of the cooperation mechanism, the profit generated by each of the two platforms improves. For test case (4), the cooperation mechanism leads to a reduction in passenger cancellations on both platforms. Whereas for test cases (5) and (6), the passenger cancellations increase slightly on platform 1, while decreasing significantly for platform 2. This is due to the fact that platform 1 is over-supplied and platform 2 is under-supplied in these two test cases, and the implementation of the cooperation mechanism led to the sharing of capacity from platform 1 to platform 2.

4.3.2. Properties of the cooperation mechanism

For test cases (4) - (6), when the cooperation mechanism is in place, we document the number of total successful passenger referrals (vehicle leases) conducted by the two platforms together, the average passenger referral (vehicle lease) profit, and the total passenger referral (vehicle lease) profit. Additionally, we also document the total profit generated by the cooperation mechanism from the two platforms, and the proportion of cooperation profit relative to total profit. These statistics are shown in Table 11.

It can be observed that as the asymmetry intensifies (test case (4) to test case (6)), the number of successful passenger referrals and vehicle leases increases, while the average referral and lease prices reduce. It suggests that as the supply and demand imbalance increases, the proposed cooperation strategy actively tries to bridge the gap. Furthermore, as each platform has more misplaced passengers or vehicles, the cooperation strategy causes the associated prices of the referrals and leases to drop, such that more successful matches can be facilitated.

Using test case (6) as a specific example, we visualize each of the two platforms' successful passenger referrals and vehicle leases, and the average profit per instance, over time, which is shown in Fig. 4.

We can observe that platform 1 is leasing a significant amount of vehicles to platform 2 at a relatively low price. Whereas platform 2 is referring a significant number of passengers to platform 1 at a relatively low price. Conversely, platform 1 is referring a minimal amount of passengers to platform 2 at a relatively high price. While platform 2 is leasing a minimal number of vehicles to platform 1 at a relatively high price. Again, this is evidence that the proposed cooperation mechanism is performing as intended.

Table 11
Passenger referral and vehicle lease statistics for the test cases (4)–(6).

	Test case (4)	Test case (5)	Test case (6)
Total successful passenger referrals	2859	3313	4340
Average passenger referral profit ($)	1.08	1.03	0.96
Total passenger referral profit ($)	3096	3400	4277
Total successful vehicle leases	4155	4662	5761
Average vehicle leases profit ($)	1.75	1.72	1.67
Total vehicle leases profit ($)	7270	8039	9598
Total profit ($)	121 956	121 567	122 340
Total profit generated from cooperation ($)	10 366	11 439	13 875
Proportion of cooperation profit relative to total profit	8.5%	9.4%	11.3%

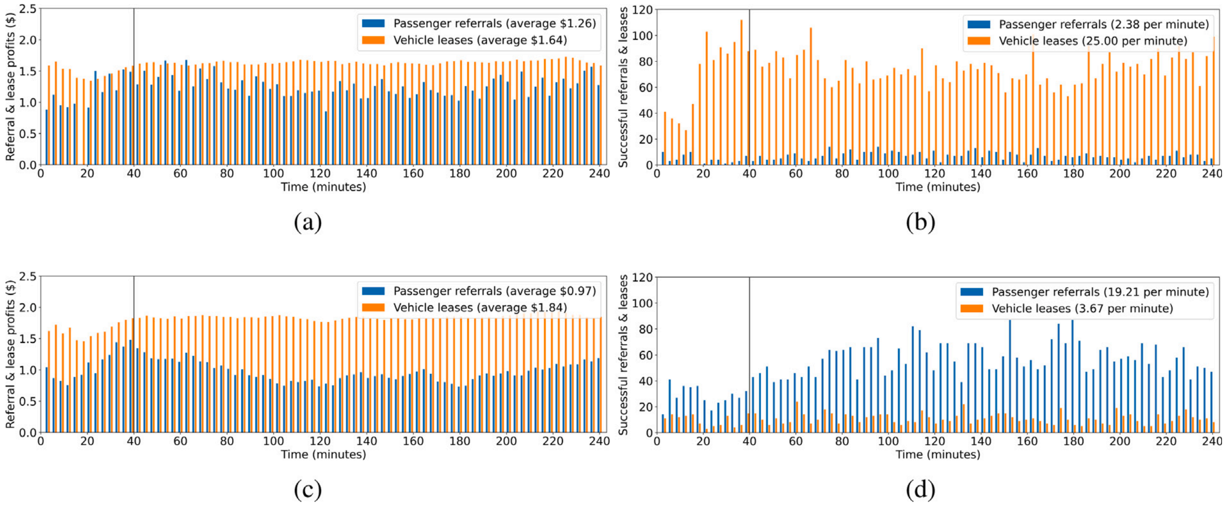

Fig. 4. The platforms' successful passenger referrals and vehicle leases, and the average profit per instance, over time, for the test case (6). The values are determined in 3-minute intervals. (a) and (b): platform 1, which is over-supplied; (c) and (d): platform 2, which is under-supplied.

Platform 1, which has an excess supply that cannot be effectively utilized, is sharing its capacity to platform 2. While platform 2, which has an excess demand that cannot be serviced effectively, is letting platform 1 provide the service. Therefore, it shows that the proposed cooperation mechanism is actively bridging the supply and demand imbalance between the two platforms, which has led to the overall performance of the duopoly.

In summary, the employment of the proposed cooperation mechanism leads to improvements in all performance indicators for all stakeholders compared to equivalent standard duopolies for all test cases considered. Additionally, the proposed cooperation mechanism is shown to be especially effective as the asymmetry between the duopoly increases. Consequently, there is no need for a policy maker or agency to intervene (Dandl et al., 2021) and the players (platforms) can/should engage in such a cooperation mechanism.

4.4. Duopoly with different prices

For the test case (7), we compare the performance indicators of the cooperation mechanism to the standard (non-cooperative) duopoly; note that there is no equivalent monopoly for this test case. The results are shown in Tables 12. The number of total successful passenger referrals (vehicle leases) conducted by the two platforms together, the average passenger referral (vehicle lease) profit, and the total passenger referral (vehicle lease) profit are shown in Table 13. Finally, each of the two platforms' successful passenger referrals and vehicle leases, and the average profit per instance, over time, are visualized in Fig. 5.

In this scenario, Platform 2 is charging passengers relatively more than Platform 1. As a result for standard duopolies, we can observe the Type II cancellations for Platform 2 is much higher than that of Platform 1, as the utilities for passengers are reduced due to higher costs. At the same time Type I cancellations for Platform 2 is reduced. This is due to the fact that there will be more idle vehicles available for Platform 2 as more passengers cancel after being matched. Overall, Platform 2 serves fewer passengers than Platform 1. Though due to the higher prices, Platform 2 generates more profit than Platform 1.

After implementing the cooperation strategy, it can be observed that key performance indicators improve. For example, the average passenger pickup times are reduced by 4.1%, the total passenger cancellations are reduced by 3.4%, while the total profit generated by the two platforms is increased by 3.5%. However, these improvements are not as significant as in previous test cases.

Table 12
Performance of the cooperation mechanism in comparison with standard asymmetrical duopoly for the test case (7). Platform 1 offers service at the 'fair' price, while Platform 2 offers service at a slightly higher price.

Performance indicators	Standard duopoly			Cooperation mechanism		
	Platform 1	Platform 2	Combined	Platform 1	Platform 2	Combined
Average passenger matching time (s)	11.2	6.8	9.0	11.0	8.3	9.6
Average passenger pickup time (s)	190.2	201.7	195.9	176.1	199.7	187.9 (−4.1%)
Average passenger in-vehicle time (s)	796.8	765.1	781.0	796.0	765.4	780.7
Type I Cancellations	6848	2142	8990	6277	2858	9135
Type II Cancellations	2907	8986	11 893	2398	8643	11 041
Total passenger cancellations	9755	11 128	20 883	8675	11 501	20 176 (−3.4%)
Number of serviced passengers	21 408	20 065	41 473	22 504	19 666	42 170
Total platform profit ($)	58 375	64 603	122 978	62 029	65 201	127 230 (3.5%)
Average vehicle idle time	13.5%	22.7%	18.1%	15.0%	19.5%	17.2%
Average vehicle dispatched time	15.2%	13.1%	14.2%	14.4%	13.2%	13.8%
Average vehicle occupied time	71.3%	64.2%	67.8%	70.7%	67.3%	69.0%
Drivers' average wage ($/hr)	24.91	22.27	23.59	24.49	23.17	23.83

Table 13
Passenger referral and vehicle lease statistics for the test case (7).

	Test case (7)
Total successful passenger referrals	2773
Average passenger referral profit ($)	1.08
Total passenger referral profit ($)	2999
Total successful vehicle leases	4567
Average vehicle leases profit ($)	1.83
Total vehicle leases profit ($)	8337
Total profit ($)	127 230
Total profit generated from cooperation ($)	11 336
Proportion of cooperation profit relative to total profit	8.9%

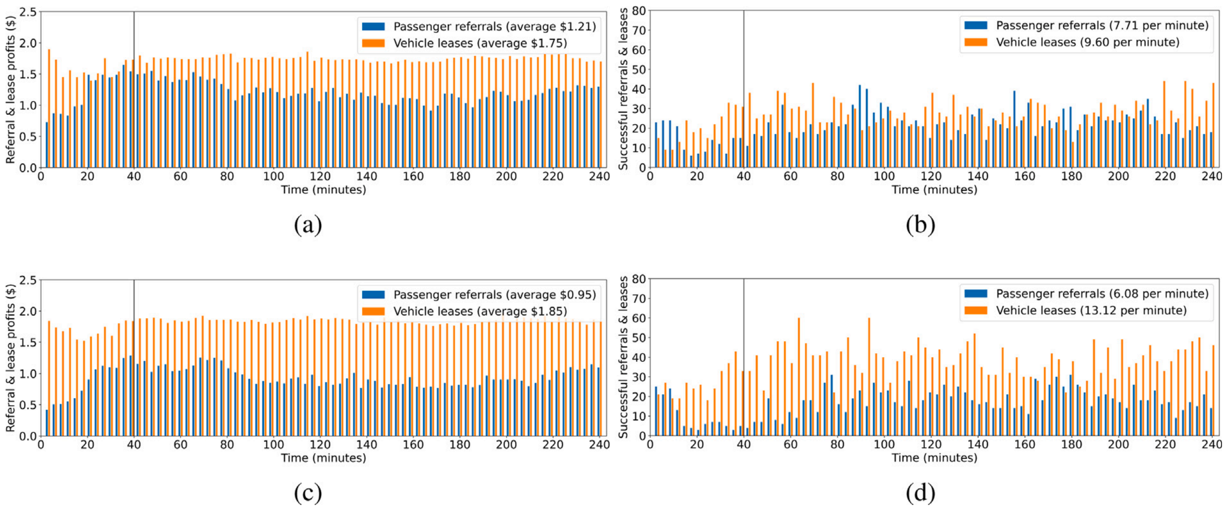

Fig. 5. Each platform's successful passenger referrals and vehicle leases, and the average profit per instance, over time. The values are determined in 3-minute intervals. (a) and (b): Platform 1 which offers service at the 'fair' price; (c) and (d): Platform 2 which offers service at a slightly higher price.

This is due to the cooperation strategy being unable to pass the benefit to the passengers. For example, passengers from Platform 2 who pay higher prices still pay the same price even though they may be serviced by Platform 1. Consequently, there will still be higher Type II cancellations for passengers from Platform 2. We can also observe this phenomenon in Fig. 3. Platform 2 is charging a relatively low passenger referral price, since the increased cancellation rate reduces the expected profit of 'holding' a passenger. However, the successful passenger referral is counter-intuitively lower for Platform 2. This is in fact due to the passengers still having to pay the original price, which leads them to Type II cancellations, and thus hindering successful referrals. Likewise, the successful vehicle leases from Platform 1 to Platform 2 are lower despite of lower lease price, due to more Type II passenger cancellations even when Platform 2 wants to utilize the leased vehicles from Platform 1.

5. Summary and future work

This paper has proposed a 'referral' cooperation mechanism under the duopoly setting, which is inspired by the capacity-sharing strategy. In its essence, whenever a platform is under-supplied while the other platform is over-supplied in a localized area, the mechanism should help the platforms to bridge their supply and demand. Under the proposed cooperation mechanism, each platform has the option to refer their passengers and let the other platform provide the service. The platforms also have the option to offer to temporarily (for a single trip) lease their vehicles for the other platform to utilize. We then present and solve the platform matching optimization problem integrating when to exercise those referral (lease) options while determining the optimal referral (lease) prices with the goal of profit maximization. Numerical experiments on a disaggregated simulator show that the proposed cooperation mechanism improves all performance indicators compared to equivalent standard duopolies. For example, symmetrical duopolies documented 4.5%–8% pickup time reduction, 2.1%–3.5% cancellation reduction for passengers; 0.5%–1.2% hourly wage increase for drivers; and 4.4%–6.1% increase in profit for platforms. Additionally, the proposed cooperation mechanism is shown to be especially effective as the asymmetry between the duopoly increases. For example, one test case considered shows the mechanism can achieve up to a 26% reduction in cancellations for passengers; a 13.8% hourly wage increase for drivers; and a 15.9% increase in profit for platforms. We argue that the effectiveness of the proposed cooperation mechanism is due to its capability to successfully bridge the supply and demand imbalances between the two platforms. Therefore as the spatial and temporal imbalance between supply and demand is bound to happen in the e-hailing market, the proposed cooperation mechanism should prove itself to be useful in reality.

This study considers the isolated effects of market fragmentation. However, a real e-hailing market can be much more dynamic. Therefore, future studies can investigate the effects of competition in the e-hailing market by relaxing some of the assumptions in this study, such as allowing passengers' and drivers' multi-homing behaviors. Furthermore, a dominant firm may wish to exert its dominance and demand a better deal from the cooperation strategy, a future study could investigate how similar strategies could play out between irrational firms. Additionally, a priority research direction is to use real data to model the cancellation behavior of customers (Type I and Type II), and to scrutinize the relationship between these cancellation types and market conditions. Finally, the policymaker may have an interest in further reducing the negative externalities of market fragmentation. Therefore, another research topic may consider the policymaker's role in further improving the proposed cooperation mechanism.

CRediT authorship contribution statement

Guipeng Jiao: Conceptualization, Data curation, Formal analysis, Investigation, Methodology, Software, Validation, Writing – original draft, Writing – review & editing. **Mohsen Ramezani:** Conceptualization, Formal analysis, Funding acquisition, Investigation, Methodology, Project administration, Supervision, Validation, Writing – original draft, Writing – review & editing.

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