Travel Time Dynamics and Applications in ITS

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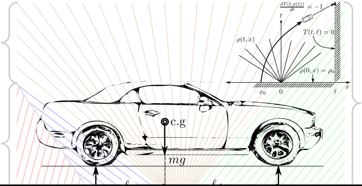
August 19^{th}, 2024



Outline I

Outline II

- Multiple Vehicles

7. Mesoscopic Model

8. Conservation
   - Mass Conservation

9. Traffic Models: 1D
   - Scalar

10. Traffic Models: 2D
    - Scalar

11. Conservation Law Solutions
    - Characteristics
    - Scalar Riemann Problems
    - Admissibility Conditions

Outline III

- Solution Properties
- Initial-Boundary Problem

12 Macroscopic Model
- Single Vehicle
- Traffic Dynamics
- Derivation of the Travel Time Dynamics

13 Transportation Networks

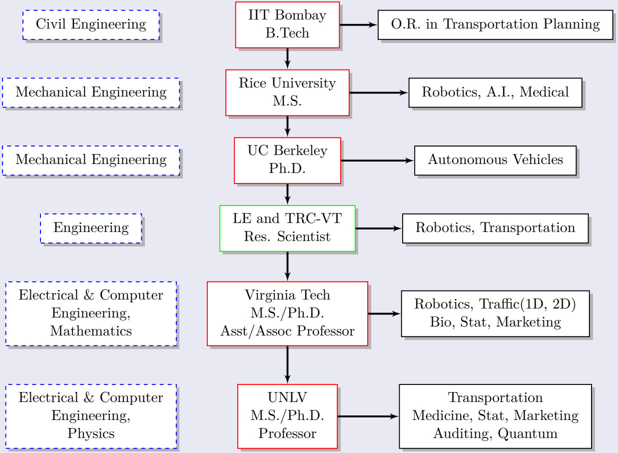
14 Heterogeneous Traffic

15 Applications

16 Conclusions

Interdisciplinary Intelligent Transportation Systems

A Personal Mathematical Journey



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Published Books

Books in Preparation

1. Vehicle Dynamics and Control
2. Algorithms, Languages, & Complexity
3. Travel Time Theory
4. Metamathematics for Quantum Logic
5. Advaita: Aham Brahamasmi, Journey through Mathematics

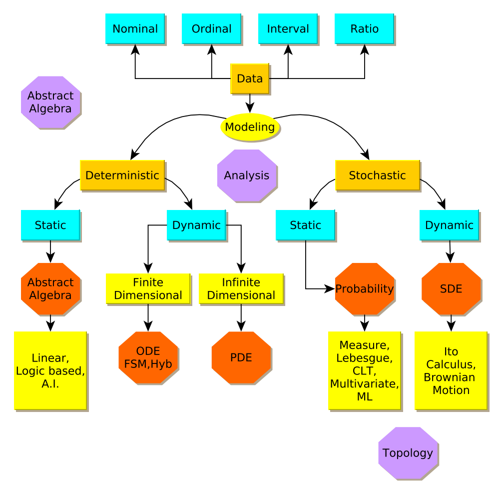
Phenomenon Modeling

Measure Based

Microscopic	Mesoscopic	Macroscopic
ODE, FDE FSM, MC	St. Mech Distributions Ito	PDE sPDE Lebesgue

Discontinuous Weak Solutions

Data and Modeling



Intelligent Transportation Systems

Technology Driven

1. Sensors
2. Processors
3. Actuators

Categories

1. Intelligent Vehicles (V2X, autonomous)
2. Traffic Management Systems
3. Traveler Information System
4. Intelligent Planning

Traffic Control

Microscopic/vehicles

1. Traction Control
2. Adaptive Cruise Control
3. Lateral Control
4. Collision Avoidance
5. V2V, V2I
6. Vehicle Streams

Macroscopic/traffic

1. Ramp Metering
2. Signalized Intersection
3. Diversion
4. Evacuation
5. Pedestrian
6. Economic Control (Tolling, congestion pricing)

Static Travel Time Model

Static Travel Time Model

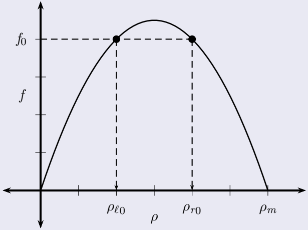
1. $T(f) = t_f \phi(f/C)$
2. BPR:
$$T(f) = t_f \left(1 + \beta \left(\frac{f}{C}\right)^{\alpha}\right)$$

Problems with Static Model

Problems with Static Model

1. Depends on uniform condition
2. Doesn't change with time
3. Based on traffic flow

Problems with Flow based Model



Microscopic Model: Single Vehicle

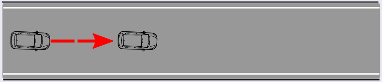
Single Vehicle

Travel Time

$$T = \frac{\ell}{v_f}$$

Microscopic Model: Two Vehicles

Two Vehicles



Travel Time

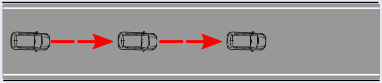
Solve:

$$\frac{dx_1}{dt} = v_f\left(1 - \frac{h_m}{x_2(t) - x_1}\right)$$

$$T = t(x_1 = \ell) - t(x_1 = 0)$$

Microscopic Model: Multiple Vehicles

Multiple Vehicles



Travel Time

Solve:

$$\frac{dx_1}{dt} = v_f\left(1 - \frac{h_m}{x_2 - x_1}\right)$$

$$\frac{dx_2}{dt} = v_f\left(1 - \frac{h_m}{x_3 - x_2}\right)$$

$$T_1 = t(x_1 = \ell) - t(x_1 = 0),\ T_2 = t(x_2 = \ell) - t(x_2 = 0)$$

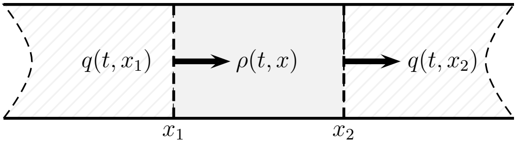
Mesoscopic Model

Multiple Vehicles

1. Statistical Mechanics
2. Speed Distribution (Boltzmann)
3. Equilibrium
4. Non-equilibrium Statistical Mechanics

Mass Conservation

Conservation 1D

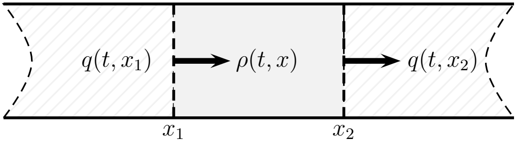


First Integral Form

$$\int_{x_1}^{x_2} \rho(t_2, x)\, dx - \int_{x_1}^{x_2} \rho(t_1, x)\, dx = \int_{t_1}^{t_2} \rho(t, x_1) v(t, x_1)\, dt - \int_{t_1}^{t_2} \rho(t, x_2) v(t, x_2)\, dt$$

Mass Conservation

Conservation 1D

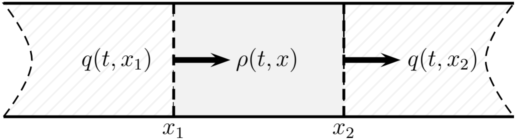


Second Integral Form

$$\frac{d}{dt}\int_{x_1}^{x_2} \rho(t,x)\,dx = \rho(t,x_1)v(t,x_1) - \rho(t,x_2)v(t,x_2)$$

Mass Conservation

Conservation 1D

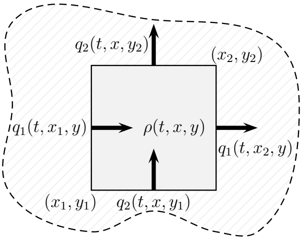


Differential Form

$$\frac{\partial}{\partial t}\rho(t, x) + \frac{\partial}{\partial x}q(t, x) = 0$$

Mass Conservation

Conservation 2D

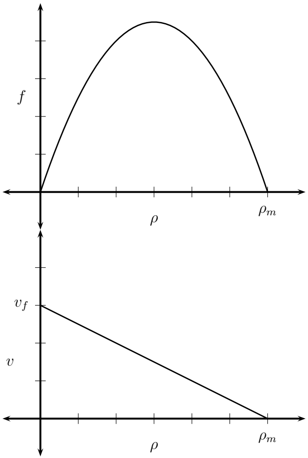


n-Dimensions

$$\frac{\partial}{\partial t}\rho(t,x) + \nabla \cdot q(t,x) = 0$$

LWR Model

Fundamental Diagram



Conservation

$$\frac{\partial}{\partial t}\rho(t,x) + \frac{\partial}{\partial x}f(t,x) = 0$$

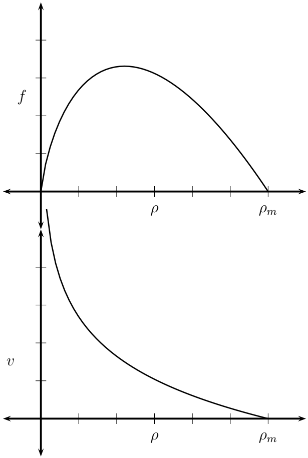
$$f = \rho(t,x)v(t,x)$$

Greenshield Model

$$v(\rho) = v_f(1 - \frac{\rho}{\rho_m})$$

LWR Model

Fundamental Diagram



Conservation

$$\frac{\partial}{\partial t}\rho(t,x) + \frac{\partial}{\partial x} f(t,x) = 0$$

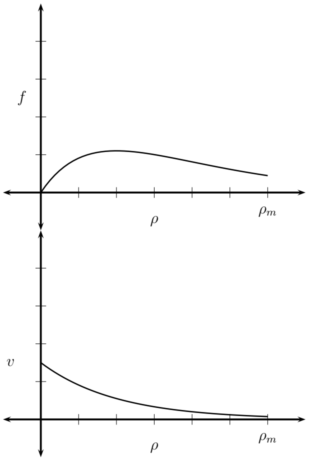
$$f = \rho(t,x) v(t,x)$$

Greenberg Model

$$v(\rho) = v_f\, ln(\frac{\rho_m}{\rho})$$

LWR Model

Fundamental Diagram



Conservation

$$\frac{\partial}{\partial t}\rho(t,x) + \frac{\partial}{\partial x}f(t,x) = 0$$

$$f = \rho(t,x)v(t,x)$$

Underwood Model

$$v(\rho) = v_f \exp(\frac{-\rho}{\rho_m})$$

LWR Model

Conservation

$$\frac{\partial}{\partial t}\rho(t,x) + \frac{\partial}{\partial x}f(t,x) = 0$$

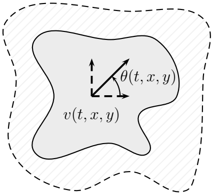
$$f = \rho(t,x)v(t,x)$$

Diffusion Model

$$v(\rho) = v_f(1 - \frac{\rho}{\rho_m}) - \frac{D}{\rho}(\frac{\partial \rho}{\partial x})$$

LWR Model

Pedestrian 2D Traffic



Model

$$\frac{\partial}{\partial t}\rho(t,x,y) + \nabla \cdot q(t,x,y) = 0$$

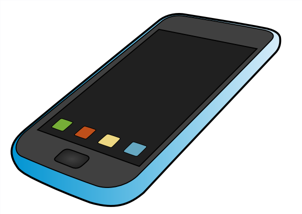
$$q = v_f\left(1 - \frac{\rho}{\rho_m}\right)\rho\begin{pmatrix} cos\theta \\ sin\theta \end{pmatrix}$$

Lagrangian Model

Model

$$\frac{\partial}{\partial t}s(t,n) + \frac{\partial}{\partial n}V(s) = 0$$

Smartphone Lagrangian Sensor

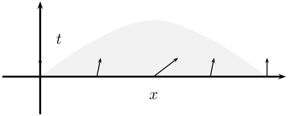


Features

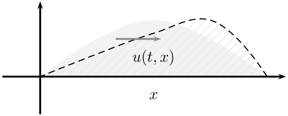
- $s(t,n)$ spacing function of time t and n vehicle number
- Modeling useful for Lagrangian sensors like smartphone
- Observability Important

Characteristics

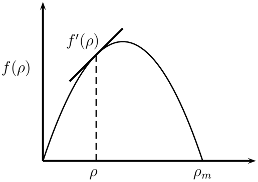
Characteristic Slopes



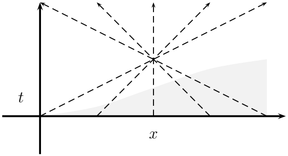
Solution at Time t



Characteristic Speed

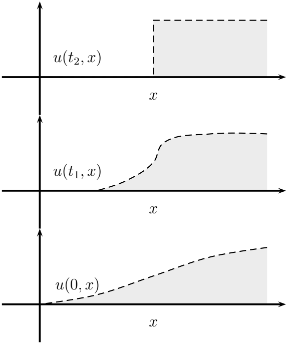


Solution Blowup



Characteristics

Initial Conditions Propagations



Distributional Solution

Cauchy Problem

$$u_t + f(u)_x = 0$$

$$u(0, x) = u_0(x)$$

Distributional Solution

A measurable locally integrable function $u(t, x)$ is a solution in the distributional sense of the Cauchy problem if for every test function ϕ

$$\iint\limits_{R^+ \times R} \left[u(t,x)\,\phi_t(t,x) + f(u(t,x))\,\phi_x(t,x)\right]\,dx\,dt + \int_R u_0(x)\,\phi(x,0)\,dx = 0$$

Weak Solution

Weak Solution

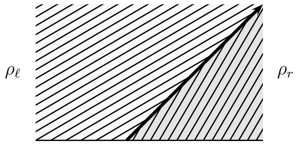
Distributional solution in the open strip; initial condition, L^1 cont. in t

$$u(t,x) = u(t,x^+)$$

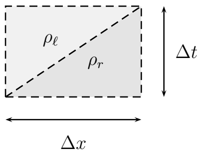
$$\lim_{t \to 0} \int_R |u(t,x) - u_0(x)| \, dx = 0$$

Shock Wave

Shock Characteristics



Shock Speed

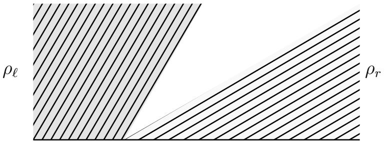


Calculating Shock Speed

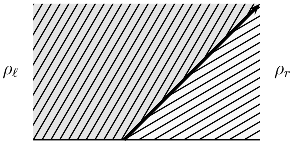
$$\lambda(\rho_r - \rho_\ell) = f(\rho_r) - f(\rho_\ell)$$

Rarefaction Wave

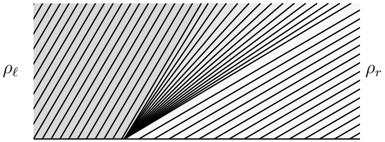
Blank Region



Entropy Violating Solution



Rarefaction Solution



Urban Morphology

Huygen's Principle

$$\|\nabla u(x,y)\|^2 = \frac{1}{s^2(x,y)}$$

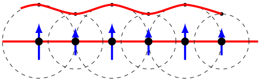
$$\frac{dx}{d\ell} = sp,$$

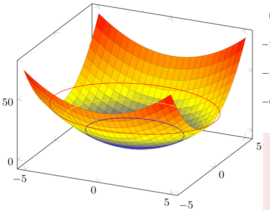
$$\frac{dy}{d\ell} = sq,$$

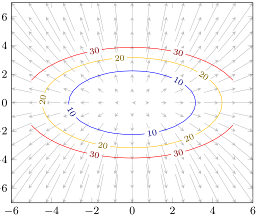
$$\frac{du}{d\ell} = \frac{1}{s},$$

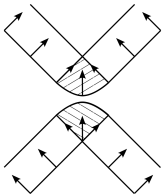
$$\frac{dp}{d\ell} = -\frac{s_x}{s^2},$$

$$\frac{dq}{d\ell} = -\frac{s_y}{s^2}$$









Admissibility Conditions

Vanishing Viscosity

$$u_t^\epsilon + f(u^\epsilon)_x = \epsilon u_{xx}^\epsilon$$

Entropy

$$\frac{\partial \eta(u)}{\partial t} + \frac{\partial q(u)}{\partial x} \leqslant 0$$

Lax Admissibility Condition

$$\lambda(u_\ell) \geqslant \lambda \geqslant \lambda(u_r)$$

Kruzkov's Entropy Function

$$\eta(u) = |u - k| \text{ and } q(u) = sign(u-k) \cdot (f(u) - f(k))$$

$$\iint_{\Pi_T} \{|u(x,t) - k|\phi_t + \mathrm{sign}(u(x,t) - k)[f(x,t,u(x,t)) - f(x,t,k)]\phi_x$$

$$-\mathrm{sign}(u(x,t) - k)[f_x(x,t,u(x,t)) - g(x,t,u(x,t))]\}\,dxdt \geqslant 0$$

$$\lim_{t \to 0} \int_{K_r} |u(x,t) - u_0(x)|\,dx = 0.$$

Solution Properties

Solution Properties

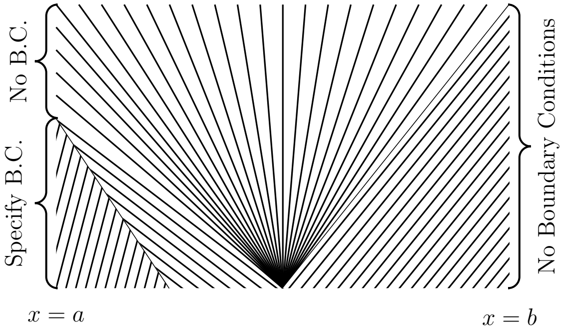
$$u_t + f(u)_x = 0\,, \quad u(0,x) = u_0(x)$$

1. *Maximum Principle*:
$$\|u(t,\cdot)\|_\infty \leqslant \|u_0\|_\infty$$
2. *Total variation diminishing*:
$$TV(u(t,\cdot)) \leqslant TV(u_0)$$
3. L^1 *Contractive*:
$$\|u(t,\cdot) - v(t,\cdot)\|_1 \leqslant \|u_0 - v_0\|_1$$
4. *Monotonicity Preserving*:
$$u_0 \text{ monotone } \Rightarrow u(t,\cdot) \text{ monotone}$$
5. *Monotonicity*:
$$u_0 \leqslant v_0 \Rightarrow u(t,\cdot) \leqslant v(t,\cdot)$$
6. *Lipschitz Continuity in time*:
$$\|u(t,\cdot) - u(s,\cdot)\|_1 \leqslant \|f\|_{Lip}\, TV(u_0)\, |t-s|$$

$\forall s,\ t \in R^+$

Initial-Boundary Problem

Initial-Boundary Problem



Macroscopic Model: Single Vehicle

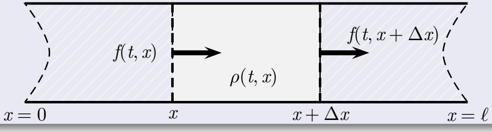
Single Vehicle

Travel Time

$$T = \frac{\ell}{v_f\left(1 - \frac{\rho}{\rho_m}\right)}$$

Macroscopic Model: Traffic Dynamics

Traffic Dynamics



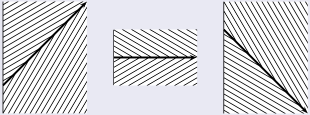
Travel Time

$$\frac{\partial}{\partial t}\rho(t,x) + \frac{\partial}{\partial x}[\rho(t,x)v(\rho(t,x))] = 0$$

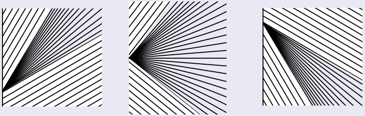
$$v(\rho(t,x)) = v_f(1 - \frac{\rho}{\rho_m})$$

Macroscopic Model: Traffic Dynamics

Shock Waves

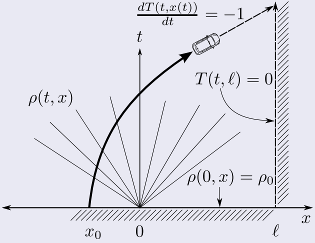


Rarefaction Waves



Derivation of the Travel Time Dynamics

Travel Time on Vehicle Path



Derivation

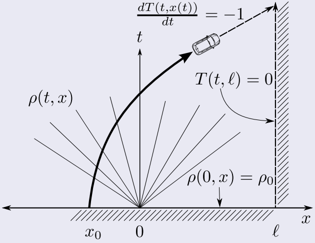
$$T(t+\Delta t, x+\Delta x) = T(t,x) - \frac{\Delta x}{v(t,x)}$$

$$\frac{\partial T(t,x)}{\partial t}\Delta t + \frac{\partial T(t,x)}{\partial x}\Delta x = -\frac{\Delta x}{v(t,x)}$$

$$\frac{\partial T(t,x)}{\partial t} + \frac{\partial T(t,x)}{\partial x} v(\rho(t,x)) + 1 = 0$$

Derivation of the Travel Time Dynamics

Travel Time on Vehicle Path



Derivation

$$\frac{dT(t,x(t))}{dt} = -1$$

$$\frac{\partial T(t,x)}{\partial t} + \frac{\partial T(t,x)}{\partial x}\frac{dx}{dt} + 1 = 0$$

$$\frac{\partial T(t,x)}{\partial t} + \frac{\partial T(t,x)}{\partial x} v(\rho(t,x)) + 1 = 0$$

Total Dynamics

Total Dynamics

$$\frac{\partial}{\partial t}\rho(t,x) + \frac{\partial}{\partial x}[\rho(t,x)v(\rho(t,x))] = 0$$

$$\frac{\partial T(t,x)}{\partial t} + \frac{\partial T(t,x)}{\partial x}v(\rho(t,x)) + 1 = 0$$

$$v(\rho(t,x)) = v_f(1 - \frac{\rho}{\rho_m})$$

Transportation Networks

Static to Dynamic

1. Conservation on links
2. Also on Nodes
3. Entropy or Vanishing Viscosity on Nodes and Links

$$\frac{\partial}{\partial t}\rho^i(t,x) + \frac{\partial}{\partial x}f(\rho^i(t,x)) = 0 \; \forall x \in [a_i, b_i], t \in [0, T]$$

$$\frac{\partial}{\partial t}\pi^i(t,x,k,r,s) + v^i(\rho^i(t,x))\frac{\partial}{\partial x}\pi^i(t,x,k,r,s) = 0 \; \forall x \in [a_i, b_i], t \in [0, T]$$

Time t, space x, link i, OD pair (r, s), and path k, fraction π.

$$\rho^i(t,x,k,r,s) = \pi^i(t,x,k,r,s)\rho^i(t,x)$$

Heterogeneous Traffic

Heterogeneous Traffic

$$f_i(\rho_1, \rho_2, \cdots, \rho_n) = \rho_i v_i(\rho_1, \rho_2, \cdots, \rho_n), \quad i = 1, 2, \cdots, n$$

Table: Multi-class Traffic Variables

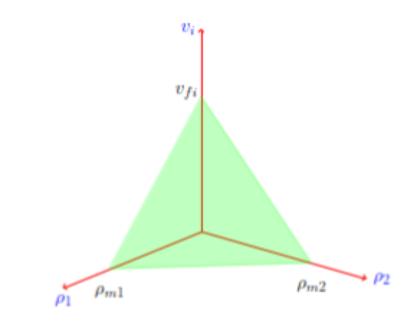
Variable	Meaning
ρ_i	Traffic density of class i
$v_i(\rho_1, \rho_2, \cdots, \rho_n)$	Traffic speed of class i as a function of all densities
$f_i(\rho_1, \rho_2, \cdots, \rho_n)$	Traffic flow of class i as a function of all densities

Heterogeneous Traffic

Heterogeneous Traffic

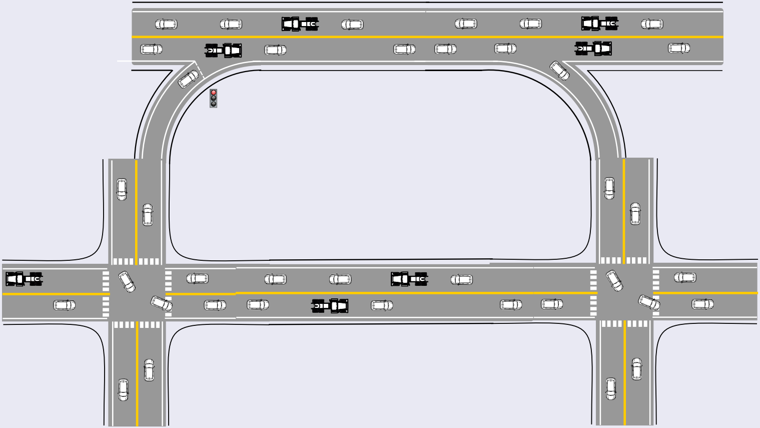
$$\forall i = 1, 2, \cdots, n \quad \frac{\partial \rho_i(t,x)}{\partial t} + \frac{\partial f_i(\rho_1(t,x), \rho_2(t,x), \cdots, \rho_n(t,x))}{\partial x} = 0,$$

$$\frac{\partial T_i(t,x)}{\partial t} + \frac{\partial T_i(\rho_1(t,x), \rho_2(t,x), \cdots, \rho_n(t,x))}{\partial x} v_i(\rho_1, \rho_2, \cdots, \rho_n) + 1 = 0$$



Applications

Applications



Conclusions

Conclusions

1. Static travel time models not adequate
2. Travel time dynamics derived from basic principles
3. Applications

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