Institute of SCIENCE TOKYO

TransportLab Seminar

2025-02-14

Multi-objective Multi-stage Stochastic Linear Programming for Strategic Planning of Shared Autonomous Vehicle Operation and Infrastructure Design with On-demand and Pre-booked Requests

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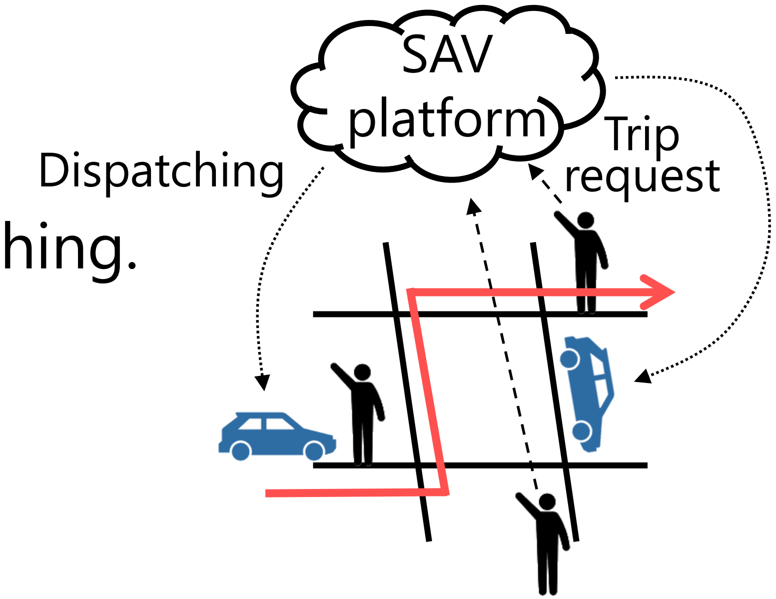
Outline

- **Introduction**
- **Problem statement**
- **Formulation**
- **Theoretical properties & solution methods**
- **Numerical experiments**
- **Summary and future works**

Kawase, R. (2024). Multi-stage stochastic linear programming for shared autonomous vehicle system operation and design with on-demand and pre-booked requests. TRB Annual Meeting 2025, Washington, D.C.

Introduction

Background: Shared Autonomous Vehicle Systems

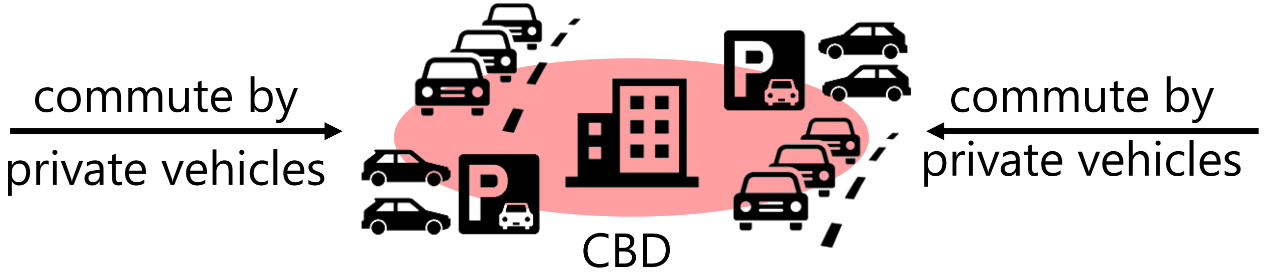


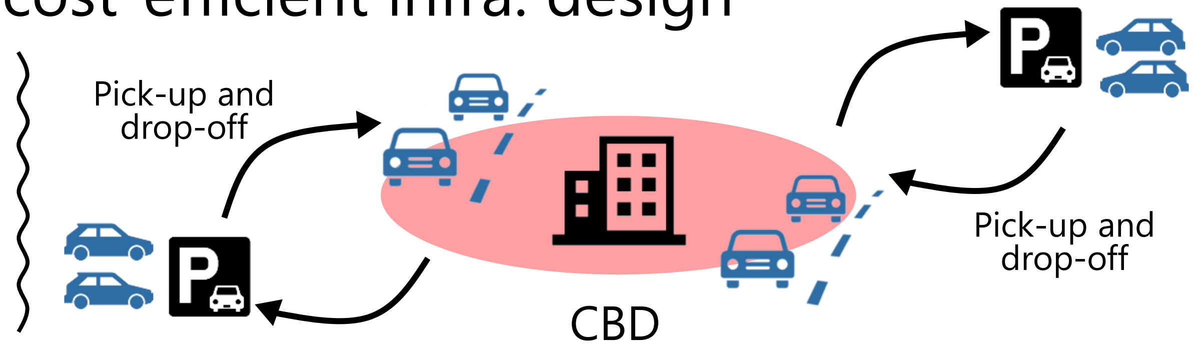
What are Shared Autonomous Vehicle (SAV) Systems?

- A large number of AVs are shared by the society.
- They provide the optimized route and ridesharing matching.

What could the SAV systems contribute to transportation planning?

- A SAV system can provide travelers with time-efficient and flexible journeys.
- planners with cost-efficient infra. design





Background: SAV Infrastructure Design

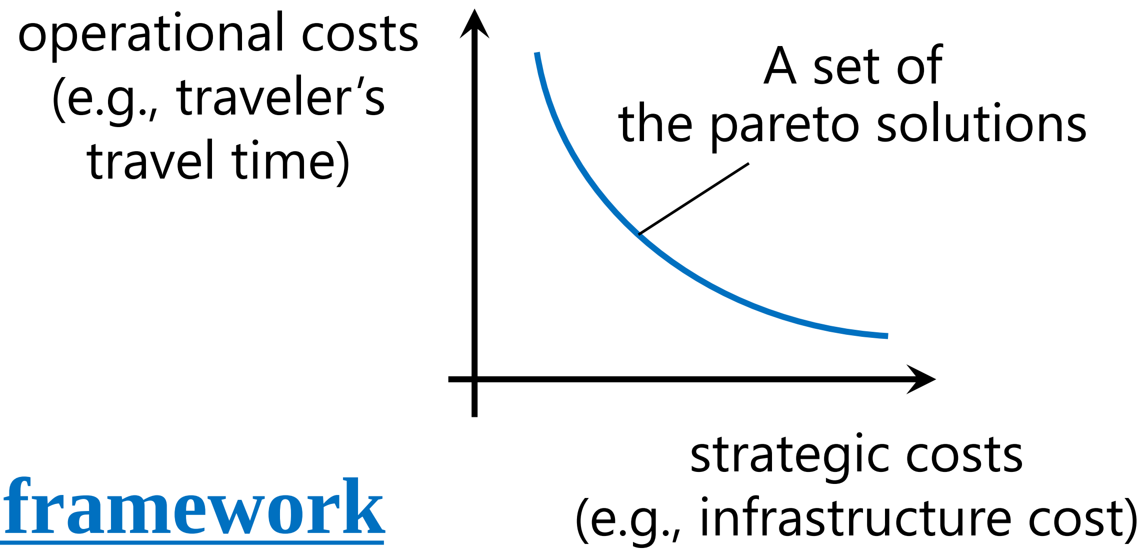
Requirements for SAV infrastructure design planning

Incorporating **SAV operations**

SAV operations	**Strategic SAV planning**
■ SAV dispatching and routing	■ Road network design
■ Vehicle-traveler assignment	■ Parking location
■ Ride-sharing matching	■ SAV fleet sizing

Trade-off relations

- Immediate response to trip requests needs sufficient fleet size of SAVs.
- Ensuring the feasibility of smooth SAV operation requires significant investments in infrastructure resources.

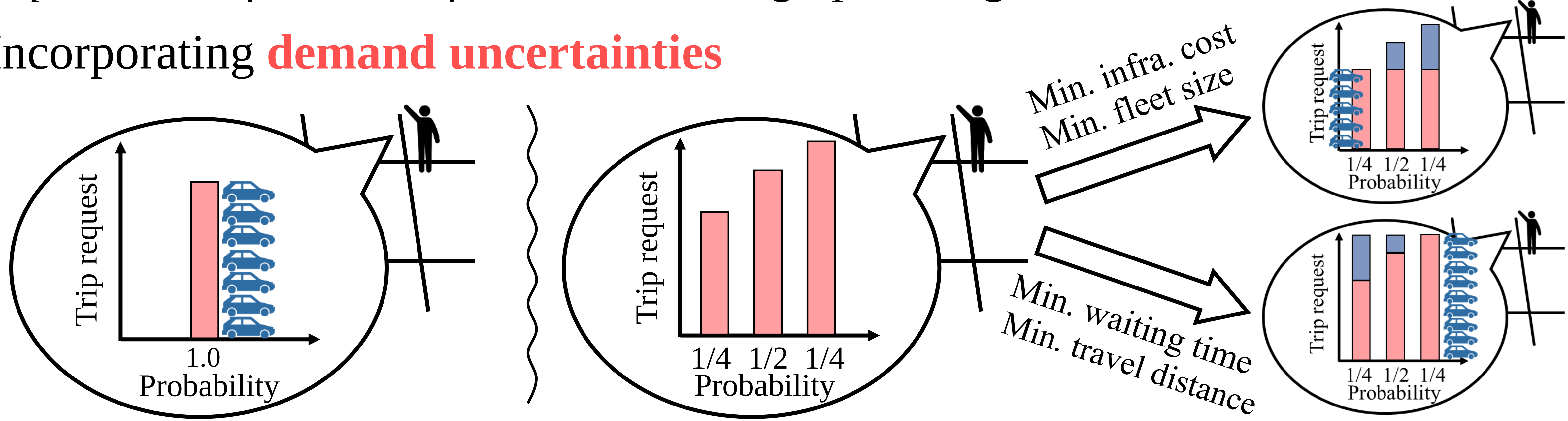


➡ **Multi-objective optimization framework**

Background: SAV Infrastructure Design

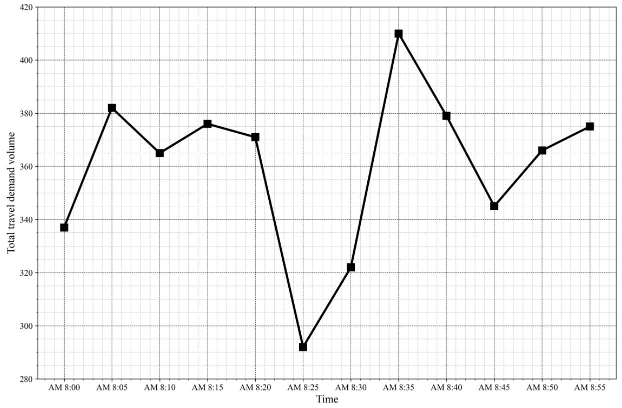
Requirements for SAV infrastructure design planning

Incorporating **demand uncertainties**



- Demand uncertainty can emphasize the importance of capturing the trade-off relations.
- Future trip requests can only be predicted stochastically.
- Trip requests are time-varying.

➡ **(Multi-stage) stochastic programming**



NYC-taxi trip data

Promising Management Strategies for Uncertain Demand

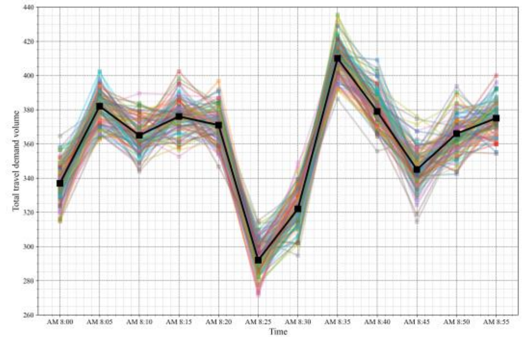
On-demand and pre-booked trip requests

<On-demand requests>

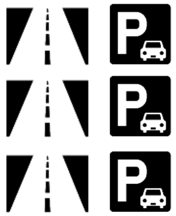
- Travelers send their trip requests (including origin and destination) at their desired departure time.

<Pre-booked requests>

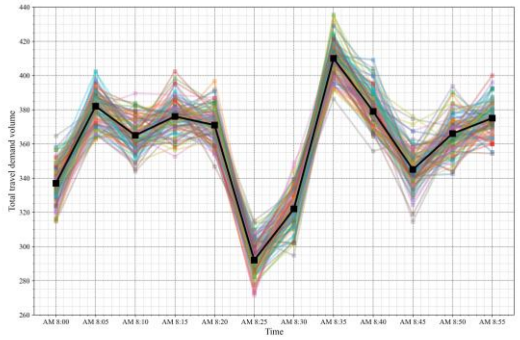
- Travelers send their trip requests (including origin, destination, **desired departure time**) **in advance**.



Decisions

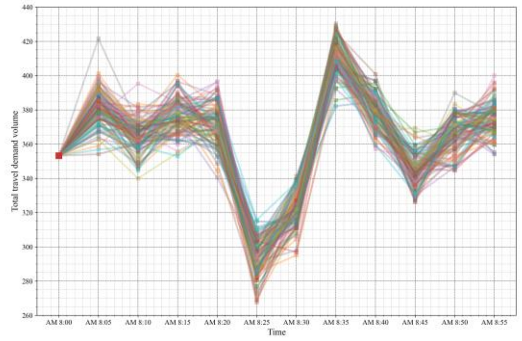


[Infra. planning stage]



Decisions

[Fleet-sizing stage]



[Operation stage]

Promising Management Strategies for Uncertain Demand

On-demand and pre-booked trip requests

<On-demand requests>

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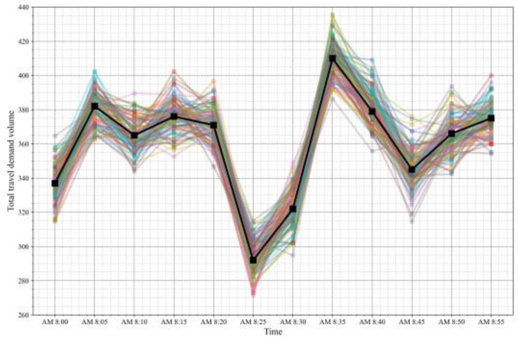

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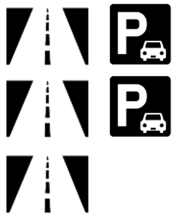

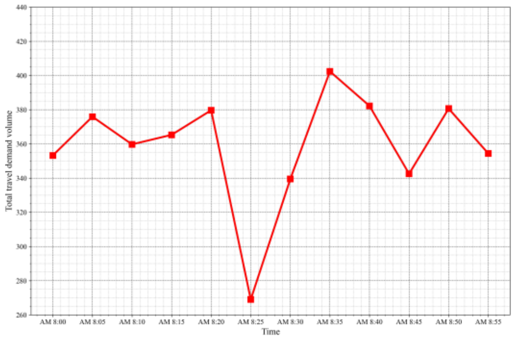

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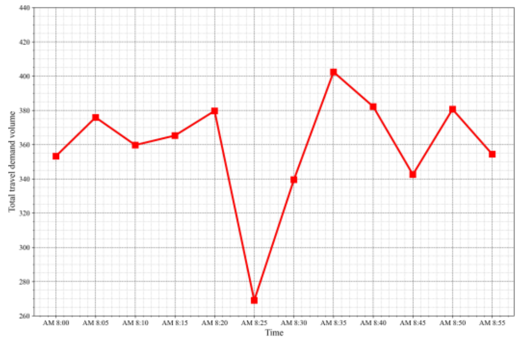


How much will be needed?

[Infra. planning stage]

How many will be needed?

[Fleet-sizing stage]

How should be operated?

[Operation stage]

→ **Infrastructure design incorporating operations with on-demand and pre-booked requests**

Methodological Challenges

Previous studies on stochastic & dynamic optimization

e.g., Reinforcement Learning, Approximate Dynamic Programming, or Customized heuristic algorithm

<Strong points>

- Address large-scale optimization problems.
- Finds approximate solutions in real time.

<Weak points>

- Not generally have optimality guarantees.
- Hard to find (general) policy implications.

Strategic-level and operational-level decision-making

- Real-time (operational-level) decision-making requires approaches to **find good solutions quickly**. Finding better solutions is all that is required.
- Strategic-level decision making (e.g., infrastructure planning) requires careful consideration, i.e., **finding elegant solutions**.

  (e.g., global optimality, explainability, and generality)

  - Infrastructure investment is expensive.
  - Infrastructure construction is irreversible. Flexible changes are difficult.
  - Accurate estimation of all model parameters is impossible.

Research Objective

Research Objective

- For infrastructure design incorporating SAV operations with uncertain on-demand and pre-booked requests,
  - we develop an optimization framework **with optimality guarantees**, and
  - we derive **theoretical properties** of the problem.

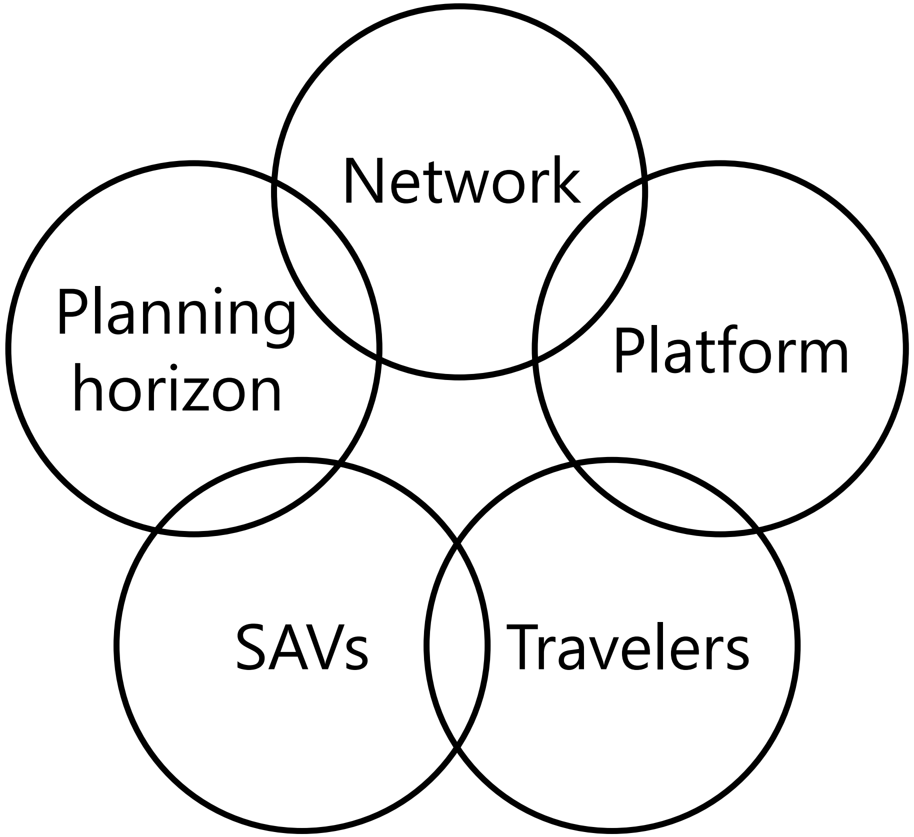
Contributions

- We formulate multi-stage stochastic linear problems (MSSLPs).
  - They can be solved by multi-stage Benders decomposition with guaranteed convergence.
  - Multi-objective optimization can be transformed into weighted sum single-objective one.
- We derive theoretical properties leveraging the linearity of the problem.
  - Ride-sharing does not increase expected strategic and operational costs, simultaneously.
  - Pre-booking does not increase expected strategic and operational costs, simultaneously.

The introduction of ride-sharing or pre-booking always leads to a Pareto-efficient SAV system.

Problem Statement

System Specification



Network

- Directed graph (e.g., Road network).

Planning horizon

- Discrete and finite.

Platform

- Centralized (single) decision-maker,
- Decides infra. design, fleet size, and operation.

SAVs

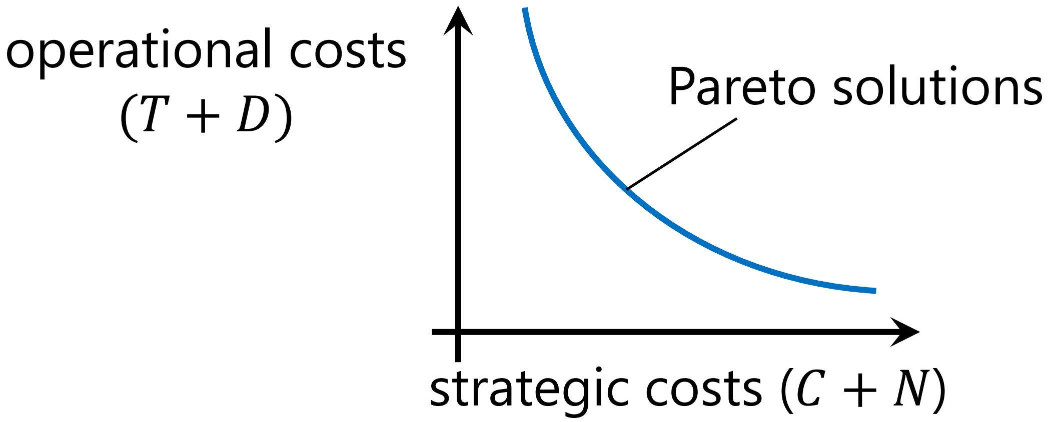
- Travel on the road network in an optimized manner.

Travelers

- Travel only by SAVs

System Specification: Platform

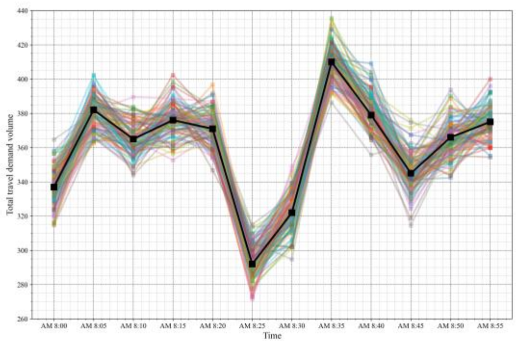
- (Continuous) decision variables
  - Node and link capacities ← Infrastructure planning
  - The number of SAVs ← Fleet sizing
  - SAV and traveler dynamic flows ← Routing and ride-sharing matching
- Objective functions: multi-objective optimization problem
  - Total travel time of travelers T
  - Total distance traveled by SAVs D
  - The number of SAVs N
  - Total infrastructure costs C



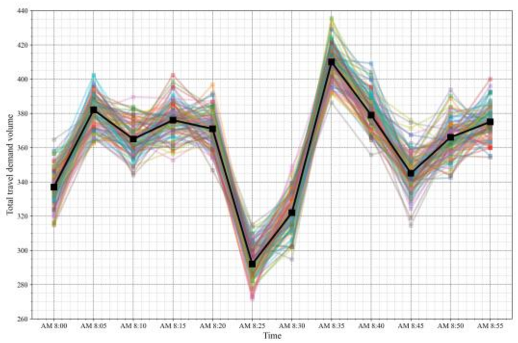
- **Demand information**
  - **Probability distributions are available when infrastructure planning,**
  - **Pre-booked trip requests are available when fleet sizing, and**
  - **On-demand trip requests are available when SAV routing and matching.**

System Specification: Planning Horizon

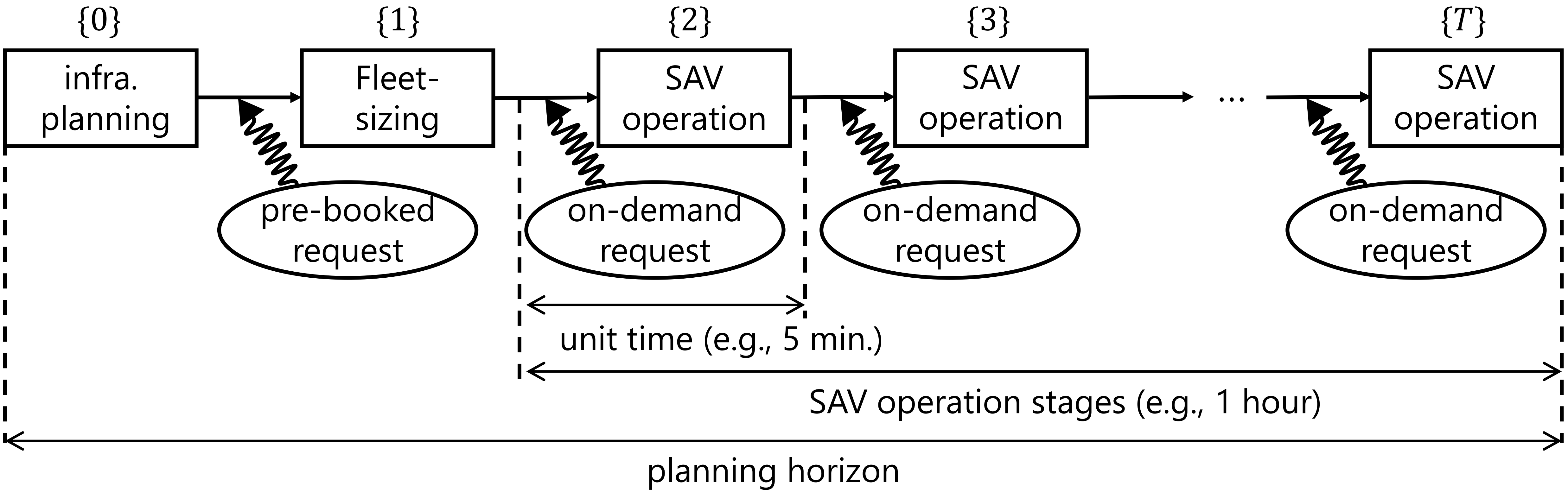
- Planning horizon: $\mathcal{T}_0 = \{0, \dots, T\}$
  - Infrastructure planning stage: $\{0\}$
  - Fleet-sizing stage: $\{1\}$
  - SAV operation stages: $\mathcal{T} = \{2, \dots, T\}$



Pre-booked requests



On-demand requests



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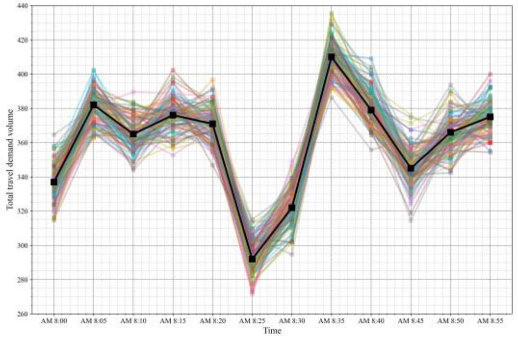
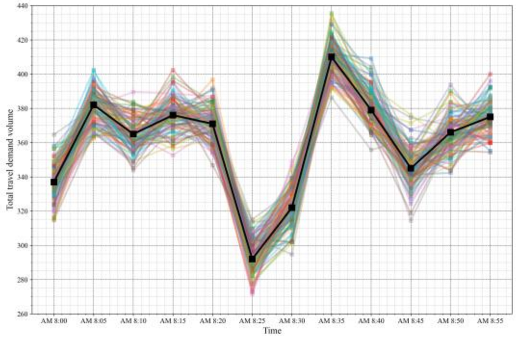
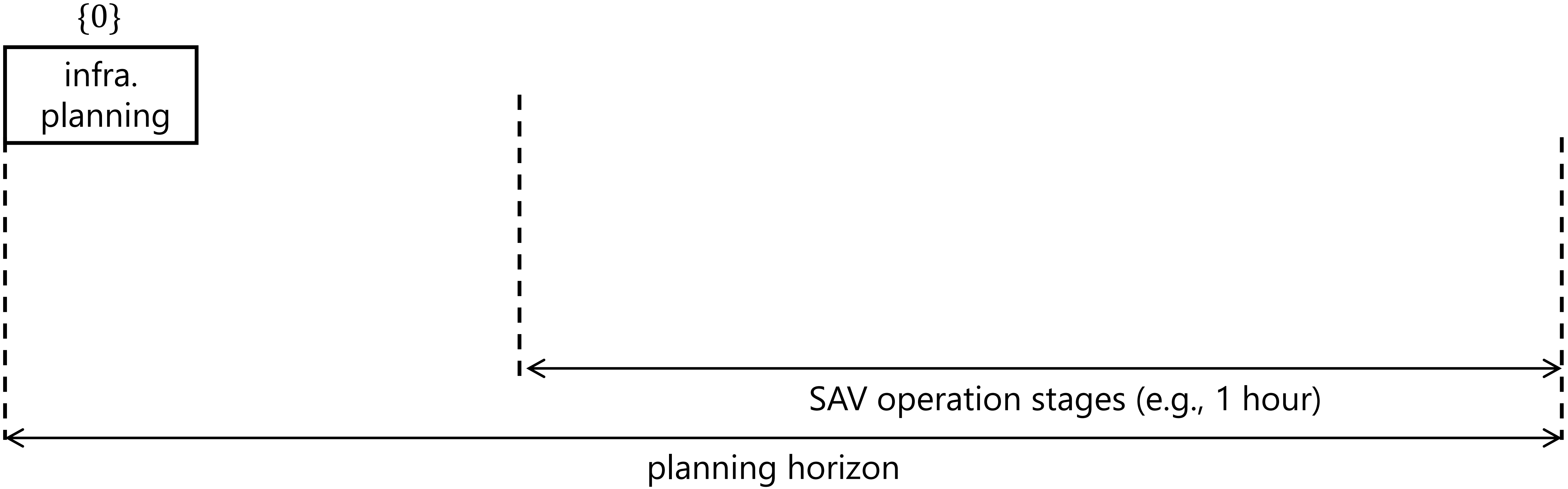

Pre-booked requests


On-demand requests



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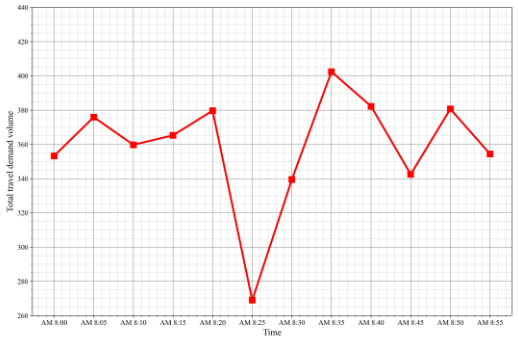
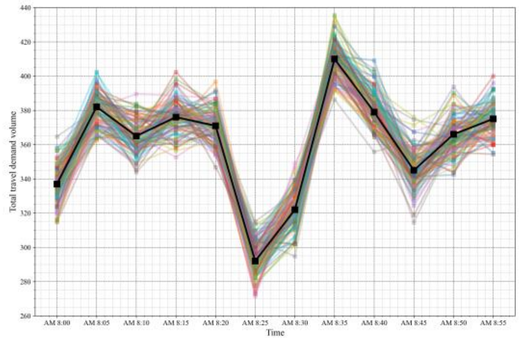
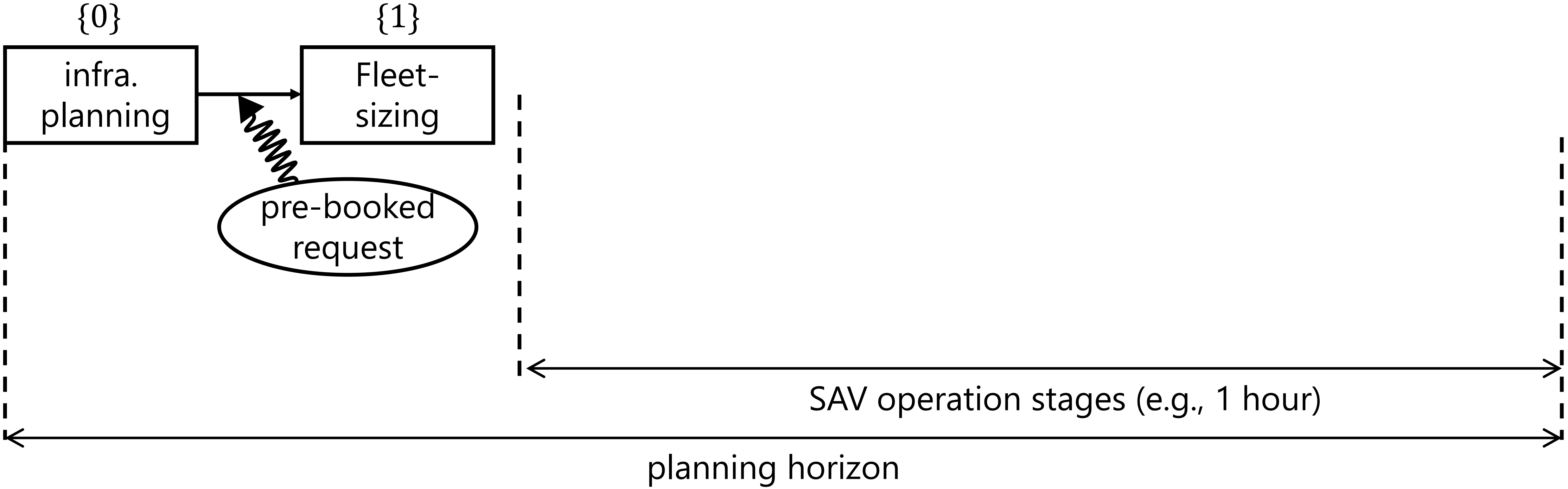


Pre-booked requests



On-demand requests



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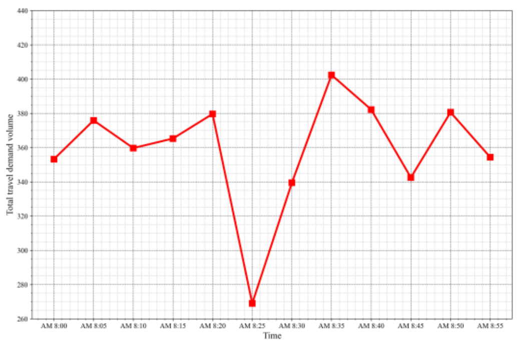
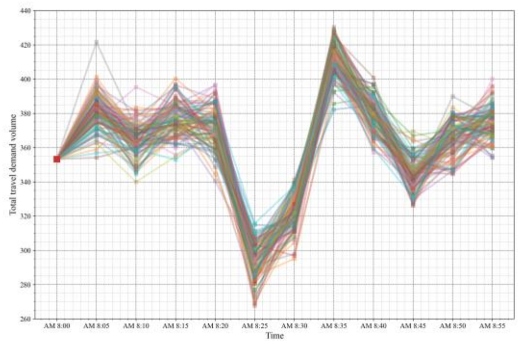
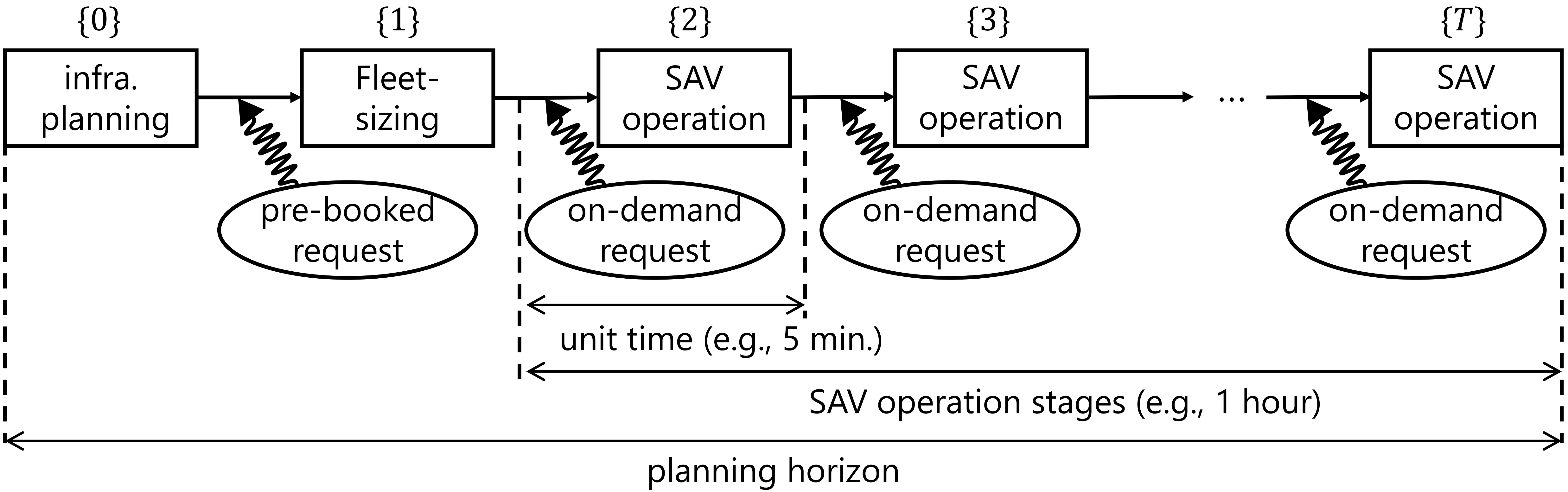


Pre-booked requests



On-demand requests



System Specification: Travelers & SAVs

Travelers

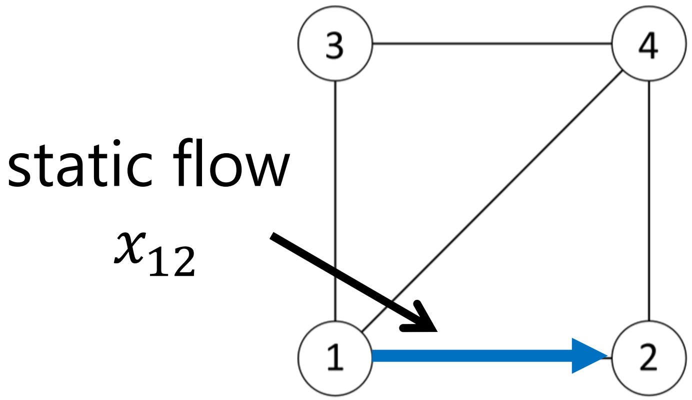
- Travelers send trip requests to SAV platforms.
  - Each request includes origin, destination, departure time, and latest arrival time.
- Probability distributions are given, and the sample space is finite.
- Requests are satisfied only by SAVs.
- Travelers follow optimized routes indicated by the platform.

SAVs

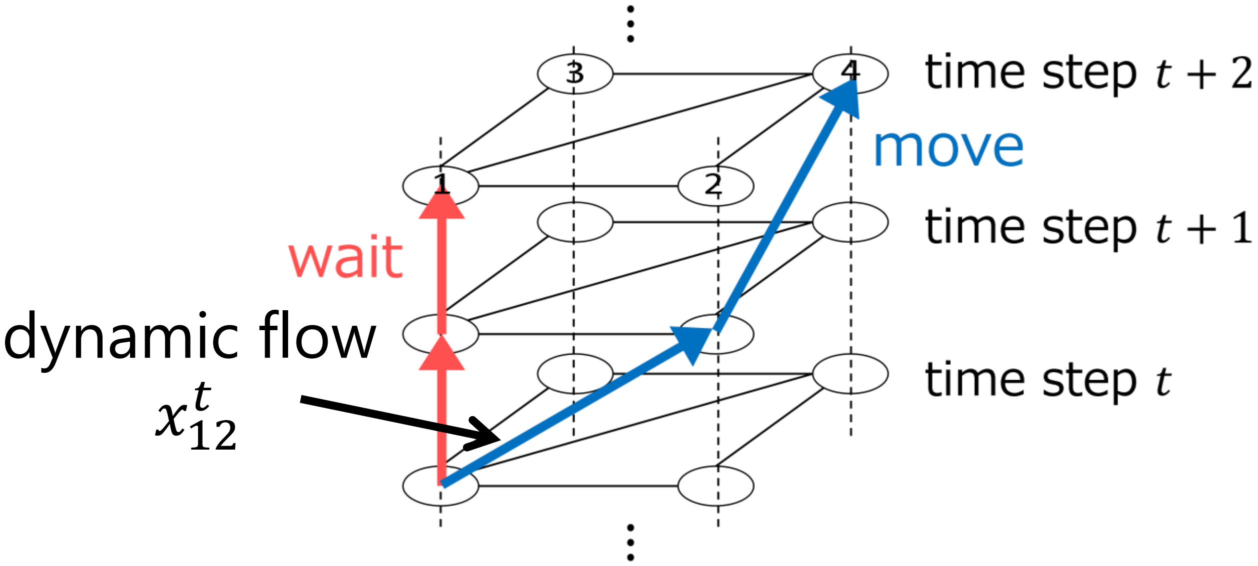
- SAVs provide transportation services to travelers with ridesharing.
- The capacity of each SAV is pre-determined and homogenous.
- SAVs run on a road network under link and node capacity constraints.
  - Links have traffic capacity and free-flow travel time.
  - Nodes have traffic (storage) capacity.
- SAVs follow the optimized route indicated by the platform.

Formulation

System Specification: Time-expanded Network



A road network



A time-expanded network

Flow constraints (examples)

Flow conservation laws $\quad \sum_j x_{ji}^{t-1} = \sum_j x_{ij}^t$

Link/node capacity constraints $\quad x_{ij}^t \leq \mu_{ij}$

Vehicle capacity constraints $\quad \sum_{k,s} (y_{s,ij}^{k,t} + \hat{y}_{s,ij}^{k,t}) \leq \rho x_{ij}^t$

x_{ij}^t	SAV flow
$y_{s,ij}^{k,t}$	Pre-booked traveler flow
$\hat{y}_{s,ij}^{k,t}$	On-demand traveler flow
k	Departure time index
s	Destination node index
μ_{ij}	Traffic capacity
ρ	Vehicle capacity (given)

Formulation: Notations

Routing and ridesharing matching for SAVs and travelers

List of variable notations

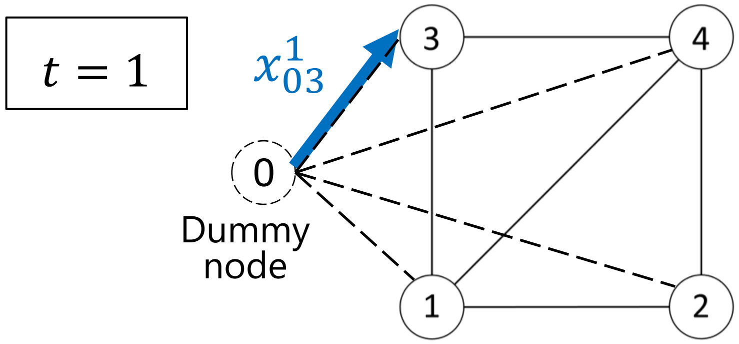
notation	definition
SAV flow → x_{ij}^t	flow of SAVs that start traveling link $ij \in \mathcal{L}$ on time step $t \in \mathcal{T}$
Traveler flow → $y_{s,ij}^{k,t}, \hat{y}_{s,ij}^{k,t}$	flow of pre-booked/on-demand travelers who start traveling link $ij \in \mathcal{L}$ on time step $t \in \mathcal{T}^k$, destination node $s \in \mathcal{S}$, and departure time step $k \in \mathcal{K}$
$A_{rs}^{k,t}, \hat{A}_{rs}^{k,t}$	Cumulative number of pre-booked/on-demand traveler departures on time step $t \in \mathcal{T}^k$, with origin node $r \in \mathcal{R}$, destination node $s \in \mathcal{S}$, and departure time step $k \in \mathcal{K}$
$D_s^{k,t}, \hat{D}_s^{k,t}$	Cumulative number of pre-booked/on-demand traveler arrivals on time step $t \in \mathcal{T}^k$, with destination node $s \in \mathcal{S}$, and departure time step $k \in \mathcal{K}$
Traffic capacity → μ_{ij}	traffic capacity of link $ij \in \mathcal{L}$
T^t	total travel time of travelers on time step $t \in \mathcal{T}$ (including waiting time on nodes)
D^t	total distance traveled by SAVs on time step $t \in \mathcal{T}$
The number of SAVs → N	total number of SAVs
C	total cost of infrastructure construction

Infra. planning

Fleet sizing

Formulation: Objective Function

Decision variables



Schematic diagram of the fleet size decision

Total infrastructure cost

$$C = \sum_{ij} c_{ij}\left(\mu_{ij} - \mu_{ij}^{\min}\right)$$

Fleet size

$$N = \sum_{i} x_{0i}^{1}$$

Dependent on realizations of pre-booked requests

Total travel time of travelers

$$T = \sum_{t \in [2,\ldots,T]} T^{t}$$

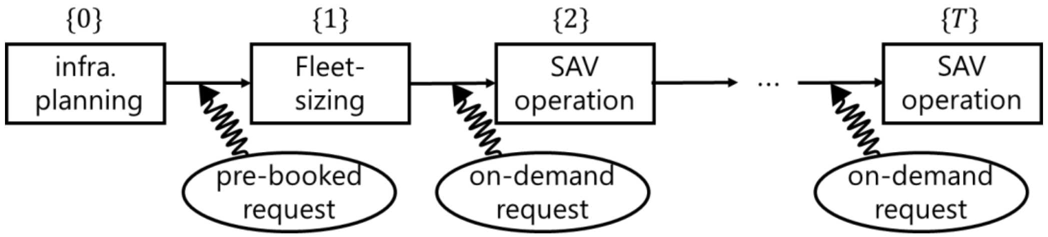
$$T^{t} = \sum_{ij,k,s} t_{ij}\left(y_{s,ij}^{k,t} + \hat{y}_{s,ij}^{k,t}\right)$$

Total distance traveled by SAVs

$$D = \sum_{t \in [2,\ldots,T]} D^{t}$$

$$D^{t} = \sum_{ij,i \neq j,k,s} d_{ij} x_{ij}^{t}$$

Dependent on realizations of on-demand requests



c_{ij}	unit cost of expanding capacity	t_{ij}	link free-flow travel time
$\mu_{ij}^{\min}$	current capacity	t_{ij}	waiting time for one time step
		d_{ij}	link travel distance

Formulation: Objective Function

Decision variables

Total infrastructure cost

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Fleet size

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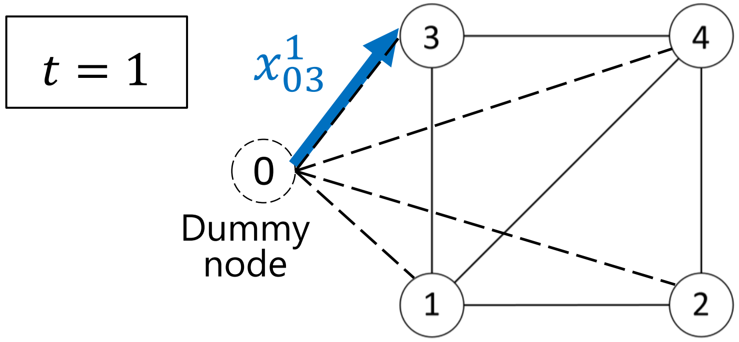

Schematic diagram of the fleet size decision

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Dependent on realizations of pre-booked requests

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$$D^{t} = \sum_{ij,i\neq j,k,s} d_{ij}x_{ij}^{t}$$

Dependent on realizations of on-demand requests

Multi-objective formulation

[MSSP-SAV]

$$\min C, \mathbb{E}[N], \mathbb{E}[T], \mathbb{E}[D]$$

$$\text{s.t.}\quad \text{constraints (e.g., capacity constraints)}$$

c_{ij}	unit cost of expanding capacity	t_{ij}	link free-flow travel time
		t_{ij}	waiting time for one time step
$\mu_{ij}^{\min}$	current capacity	d_{ij}	link travel distance

Theoretical properties & solution methods

Reformulation to Single-objective Optimization

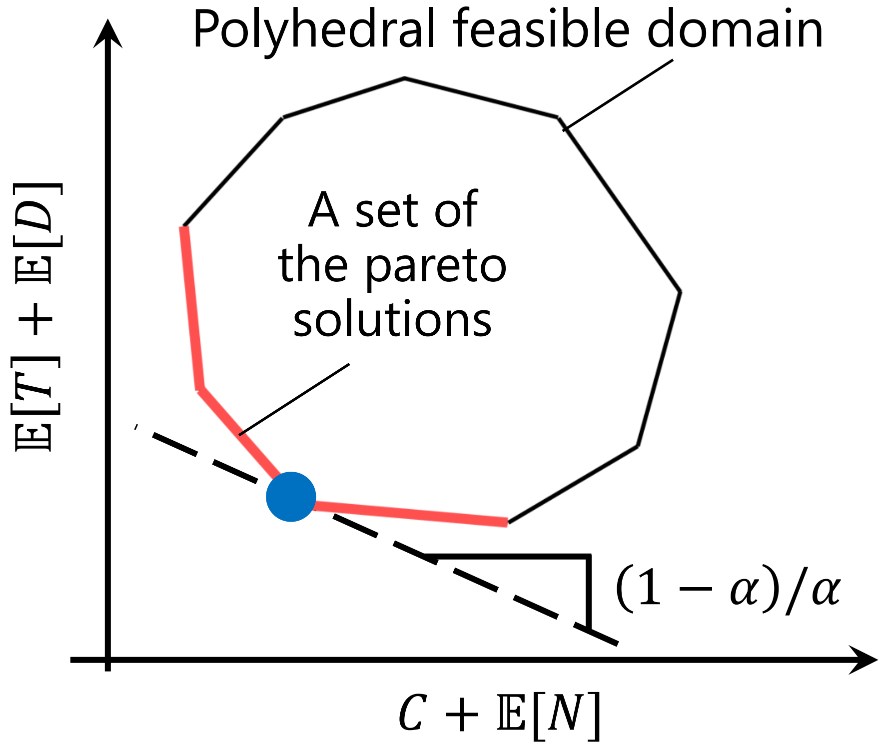
Multi-objective formulation

[MSSP-SAV]

$$\min C, \mathbb{E}[N], \mathbb{E}[T], \mathbb{E}[D]$$

s.t. linear constraints (e.g., capacity constraints)

- [MSSP-SAV] is solved when its Pareto frontier (a set of the Pareto efficient solutions) is derived.
- A solution is Pareto-efficient* when any of the objective function values cannot be decreased without increasing the other(s).



* The definition of a Pareto-efficient solution in stochastic programs can be seen in Dowson et al. (2022).

Single-objective reformulation

[MSSP-SAV-WS]

$$\min \alpha(C + \mathbb{E}[N]) + (1-\alpha)(\mathbb{E}[T] + \mathbb{E}[D])$$

s.t. linear constraints (e.g., capacity constraints)

α	weighted parameter $(0 \leq \alpha \leq 1)$
	$\alpha = 0$: minimization of $\mathbb{E}[T] + \mathbb{E}[D]$
	$\alpha = 1$: minimization of $C + \mathbb{E}[N]$

Reformulation to Dynamic Programming Equations

$\boldsymbol{z}^t$: decision variable vector
$\boldsymbol{\xi}^{[t]} = \{\boldsymbol{\xi}^1, \ldots, \boldsymbol{\xi}^t\}$: stochastic demand process
$\chi^t(\boldsymbol{z}^{t-1}, \boldsymbol{\xi}^t)$: feasible region given past decisions $\boldsymbol{z}^{t-1}$ and realizations $\boldsymbol{\xi}^t$

Nested reformulation of [MSSP-SAV-WS]

Pre-booked requests **On-demand requests**

$$\underbrace{\min_{\boldsymbol{z}^0 \in \mathcal{X}^0} F^0(\boldsymbol{z}^0)}_{\text{Infra. planning}} + \mathbb{E}^0 \left[\underbrace{\min_{\boldsymbol{z}^1 \in \mathcal{X}^1(\boldsymbol{z}^0, \boldsymbol{\xi}^{[1]})} F^1(\boldsymbol{z}^1, \boldsymbol{\xi}^{[1]})}_{\text{Fleet sizing}} + \underbrace{\mathbb{E}^1 \left[\min_{\boldsymbol{z}^2 \in \mathcal{X}^2(\boldsymbol{z}^1, \boldsymbol{\xi}^{[2]})} F^2(\boldsymbol{z}^2, \boldsymbol{\xi}^{[2]}) + \mathbb{E}^2 \left[\cdots + \mathbb{E}^{T-1} \left[\min_{\boldsymbol{z}^T \in \mathcal{X}^T(\boldsymbol{z}^{T-1}, \boldsymbol{\xi}^{[T]})} F^T(\boldsymbol{z}^T, \boldsymbol{\xi}^{[T]}) \right] \right] \right]}_{\text{SAV operations}} \right],$$

Infra. planning **Fleet sizing** **SAV operations**

where $F^t = \begin{cases} \alpha C & if\ t = 0 \\ \alpha N & if\ t = 1 \\ (1-\alpha)(T^t + D^t) & otherwise \end{cases} \quad \forall t.$

Stochastic Dual Dynamic Programming (SDDP)

- SDDP can yield the optimal solution to MSSPs* with guaranteed convergence under
  - Feasible region $\boldsymbol{\chi}^t$ is a non-empty, unbounded, and convex,
  - The objective function F^t is convex,
  - The stochastic process $\{\boldsymbol{\xi}^1, \ldots, \boldsymbol{\xi}^T\}$ is Markov, and
  - The number of realizations of $\{\boldsymbol{\xi}^1, \ldots, \boldsymbol{\xi}^T\}$ is finite.

* The sufficient condition of the global convergence can be seen in Guigues (2016) and Dowson (2020).

Theoretical properties

* We consider a probability space $(\Omega, \mathcal{F}, P)$, and we let $\{\emptyset, \Omega\} = \mathcal{F}^0 \subset \mathcal{F}^1 \subset \cdots \subset \mathcal{F}^T = \mathcal{F}$ be sub sigma-algebras of $\mathcal{F}$ that form a filtration.

Theorem 1.

For all $\hat{\rho} > \tilde{\rho} > 0$ and for all Pareto-efficient solutions in [MSSP-SAV] with $\rho = \tilde{\rho}$, there exists more weakly efficient solutions in [MSSP-SAV] with $\rho = \hat{\rho}$.

- $\rho = 1$ represents peer-to-peer matching, whereas $\rho > 1$ represents ride-share matching.
- Ride-sharing can reduce strategic and operational costs simultaneously if SAV systems are properly designed and operated.

Theorem 2*.

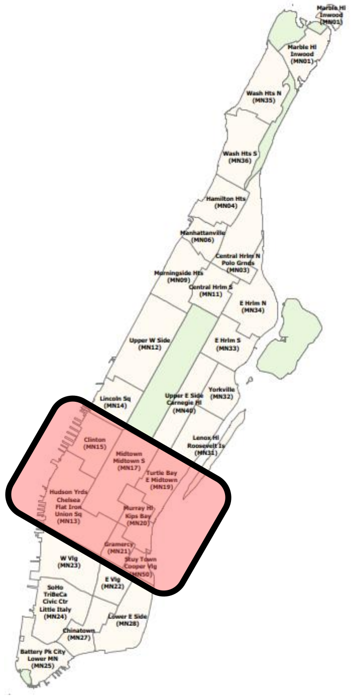
For all $\hat{\mathcal{F}}^1 \supset \tilde{\mathcal{F}}^1 \supseteq \emptyset$, where $\mathcal{F}^t$ corresponds to the information available through time t, and for all Pareto-efficient solutions in [MSSP-SAV] with $\mathcal{F}^1 = \tilde{\mathcal{F}}^1$, there exists more weakly efficient solutions in [MSSP-SAV] with $\mathcal{F}^1 = \hat{\mathcal{F}}^1$.

- $\mathcal{F}^1 = \mathcal{F}^0$ represents all trip requests are on-demand, whereas $\mathcal{F}^1 = \mathcal{F}$ represents the opposite (i.e., pre-booked).
- Pre-booking options can reduce strategic and operational costs simultaneously if SAV systems are properly designed and operated.

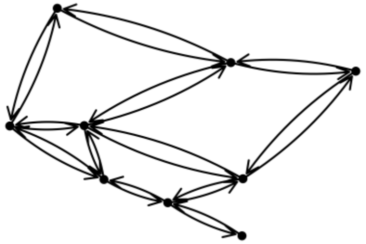
Numerical experiments

Numerical Experiments: Settings

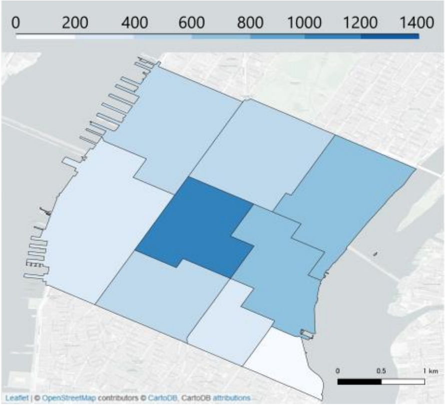
- Numerical experiments with actual travel data from New York City (NYC) were conducted.
- The NYC taxi data from 8:00 to 9:00 on 2019-04-01 (Monday) in Midtown Manhattan was inputted as expected values of travelers' demand.
- The expected total travelers' demand was 4,320.
- The proportion of pre-booked requests to passenger demand, called reserved rate p, was given as follows: $p = 0.0, 0.25, 0.5, 0.75,$ and 1.0.
- The network parameters (e.g., travel time) were set according to Seo & Asakura (2022).



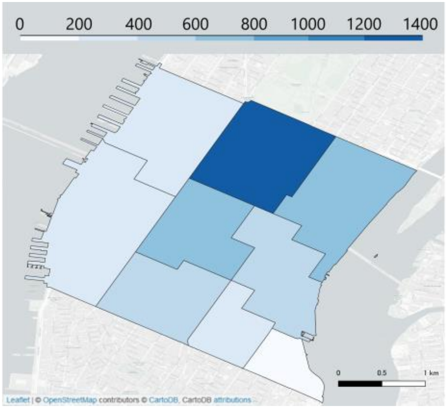
Manhattan Neighborhood Tabulation Areas



Network

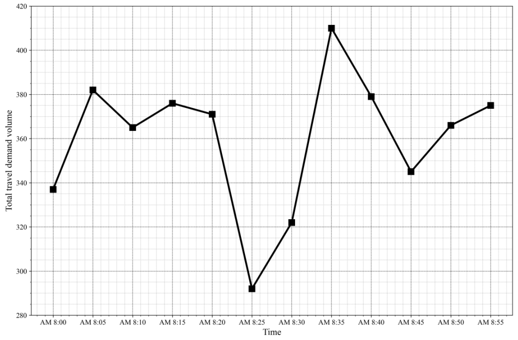


Generation demand

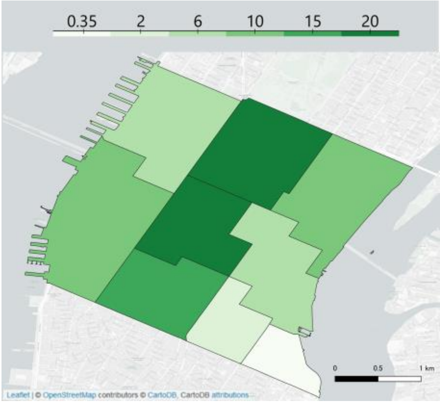


Attraction demand

Travelers' demand (generated from the NYC taxi data)



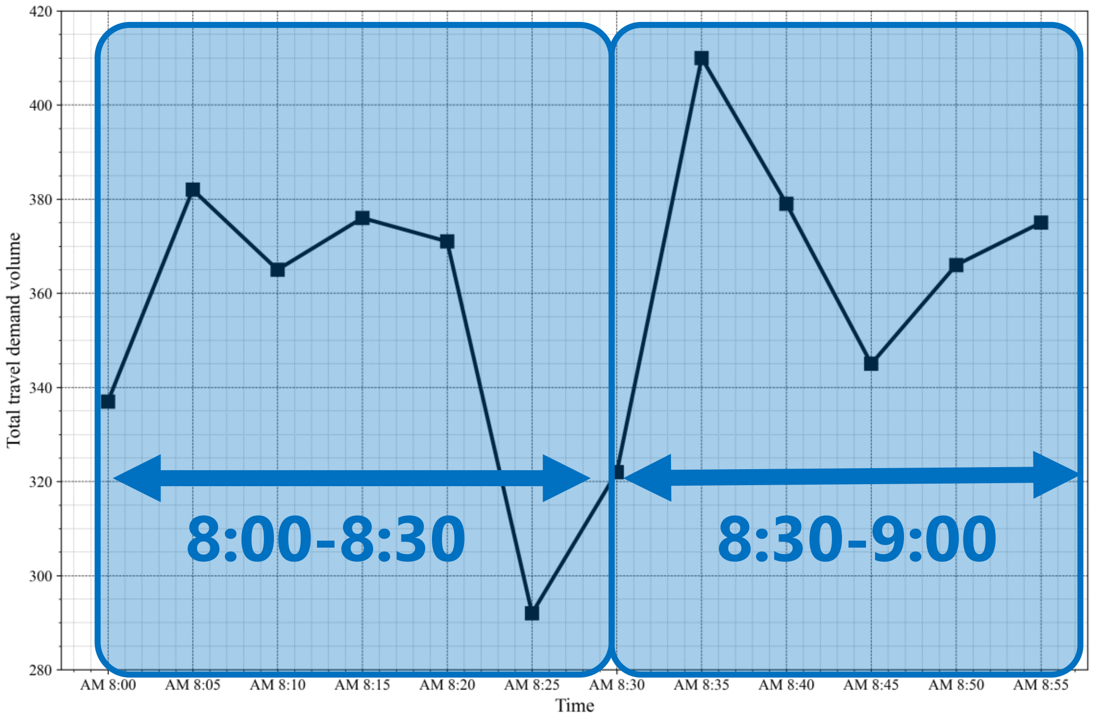
Time-dependent Travelers' demand (generated from the NYC taxi data)



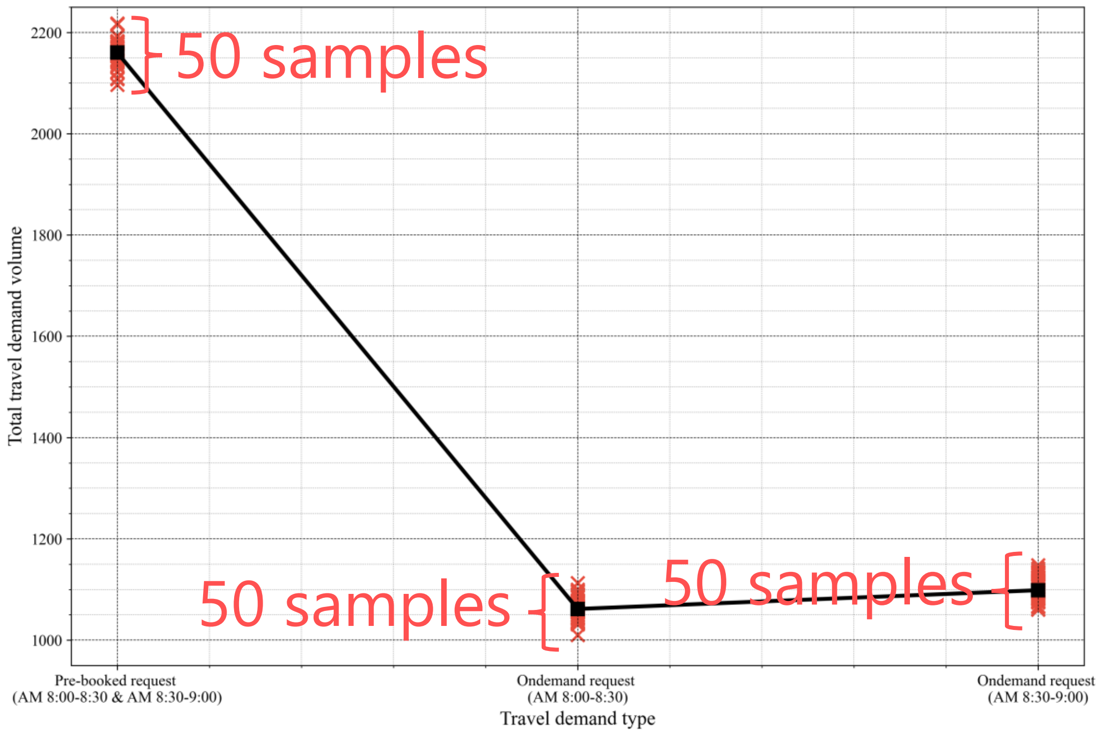
Land value
(http://www.radicalcartography.net/index.html?manhattan-value)

Numerical Experiments: Settings

- Travelers' demand was aggregated with a 30 min departure time aggregation width.
- Travelers' demand scenarios was sampled from multivariate uniform distributions.
- We considered 50 samples for pre-booked requests, 50 samples for on-demand requests (AM 8:00-8:30), and 50 samples for on-demand requests (AM 8:30-9:00). The total number of scenarios was 50^3=125,000.



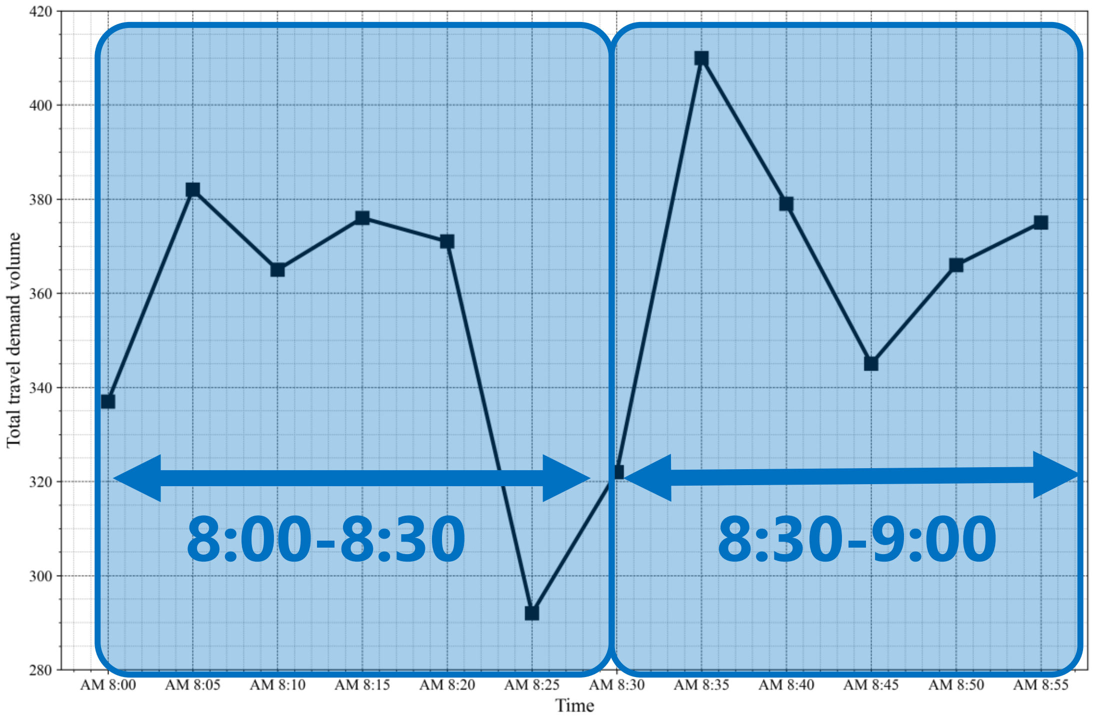
Time-dependent Travelers' demand
(generated from the NYC taxi data)



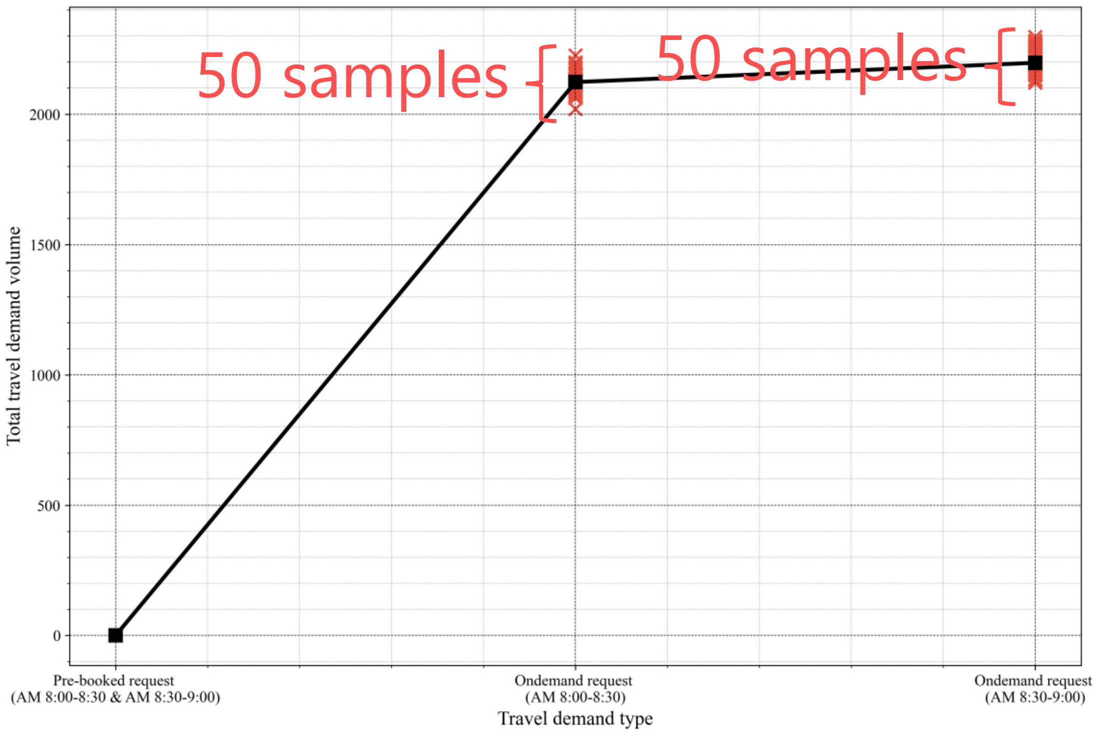
Input Travelers' demand scenarios
(example: reserved rate $p = 0.5$)

Numerical Experiments: Settings

- Travelers' demand was aggregated with a 30 min departure time aggregation width.
- Travelers' demand scenarios was sampled from multivariate uniform distributions.
- We considered 50 samples for pre-booked requests, 50 samples for on-demand requests (AM 8:00-8:30), and 50 samples for on-demand requests (AM 8:30-9:00). The total number of scenarios was 50^3=125,000.



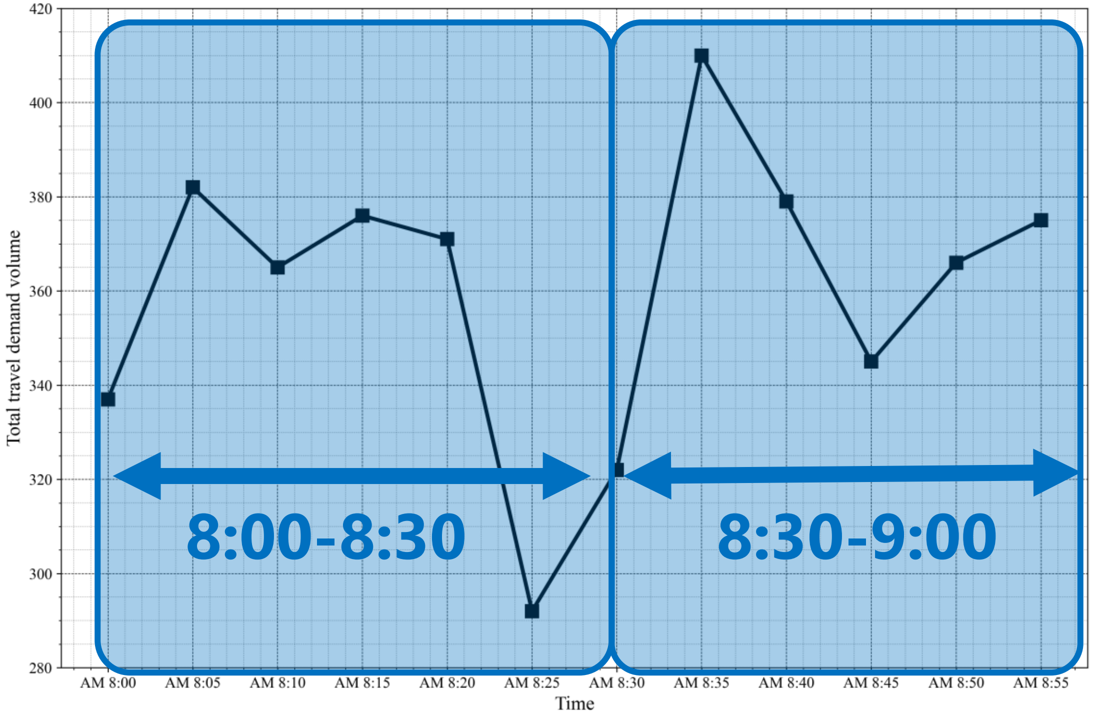
Time-dependent Travelers' demand
(generated from the NYC taxi data)



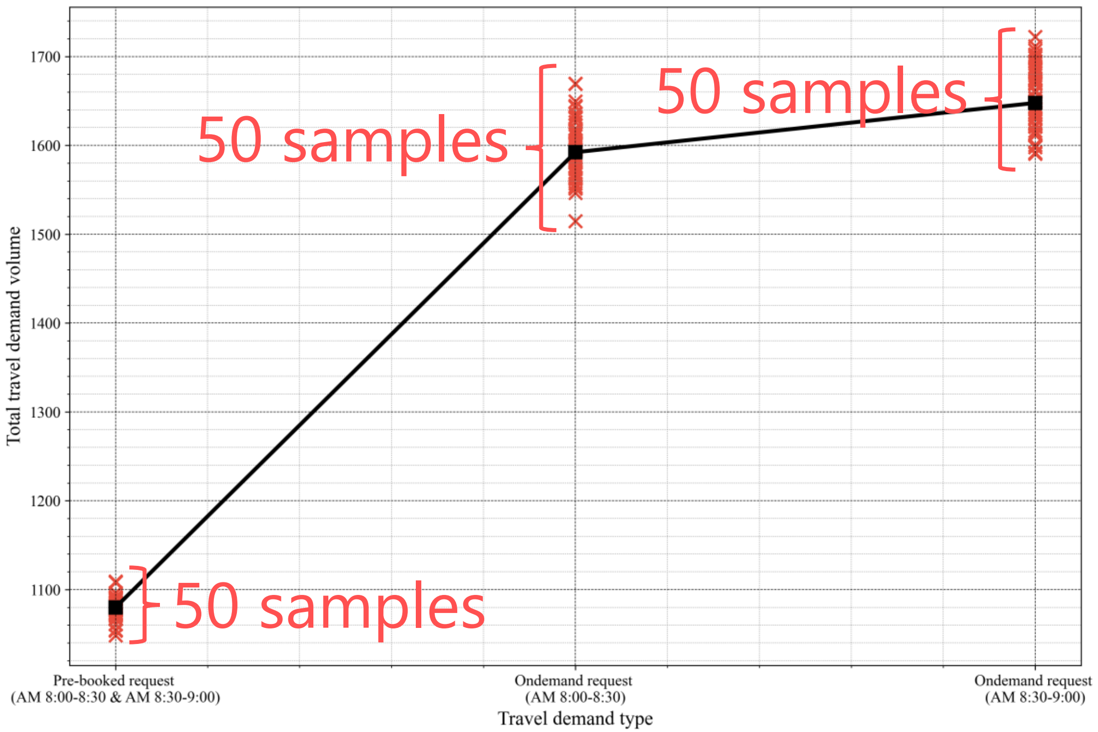
Input Travelers' demand scenarios
(example: reserved rate $p = 0.0$)

Numerical Experiments: Settings

- Travelers' demand was aggregated with a 30 min departure time aggregation width.
- Travelers' demand scenarios was sampled from multivariate uniform distributions.
- We considered 50 samples for pre-booked requests, 50 samples for on-demand requests (AM 8:00-8:30), and 50 samples for on-demand requests (AM 8:30-9:00). The total number of scenarios was 50^3=125,000.



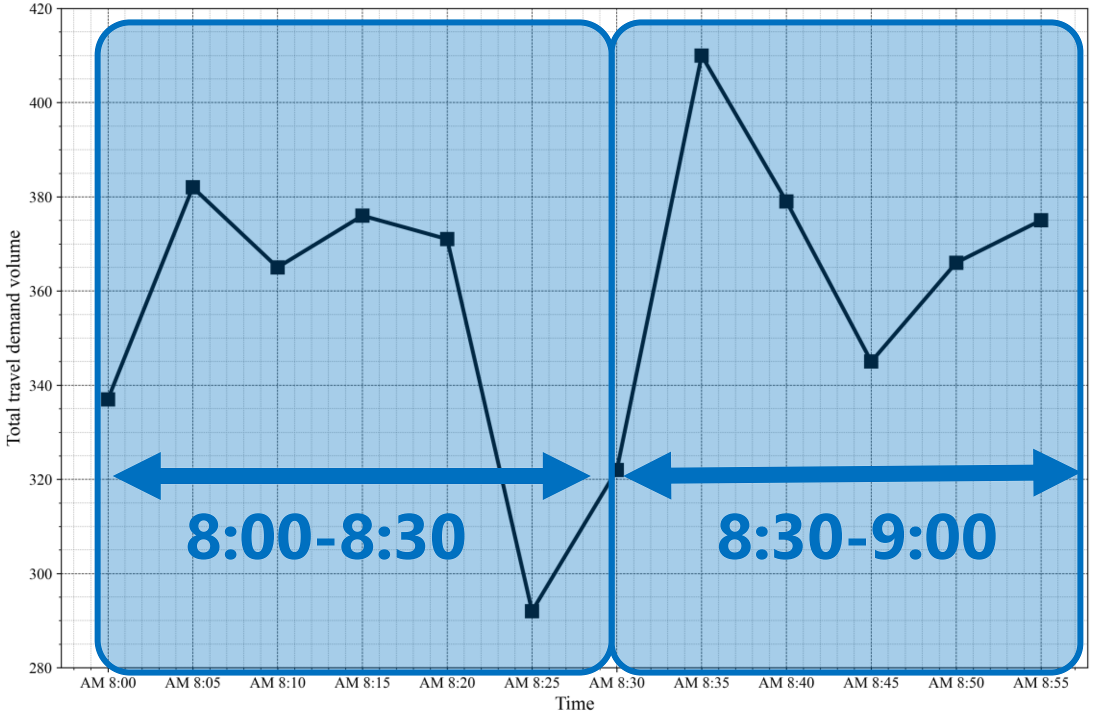
Time-dependent Travelers' demand
(generated from the NYC taxi data)



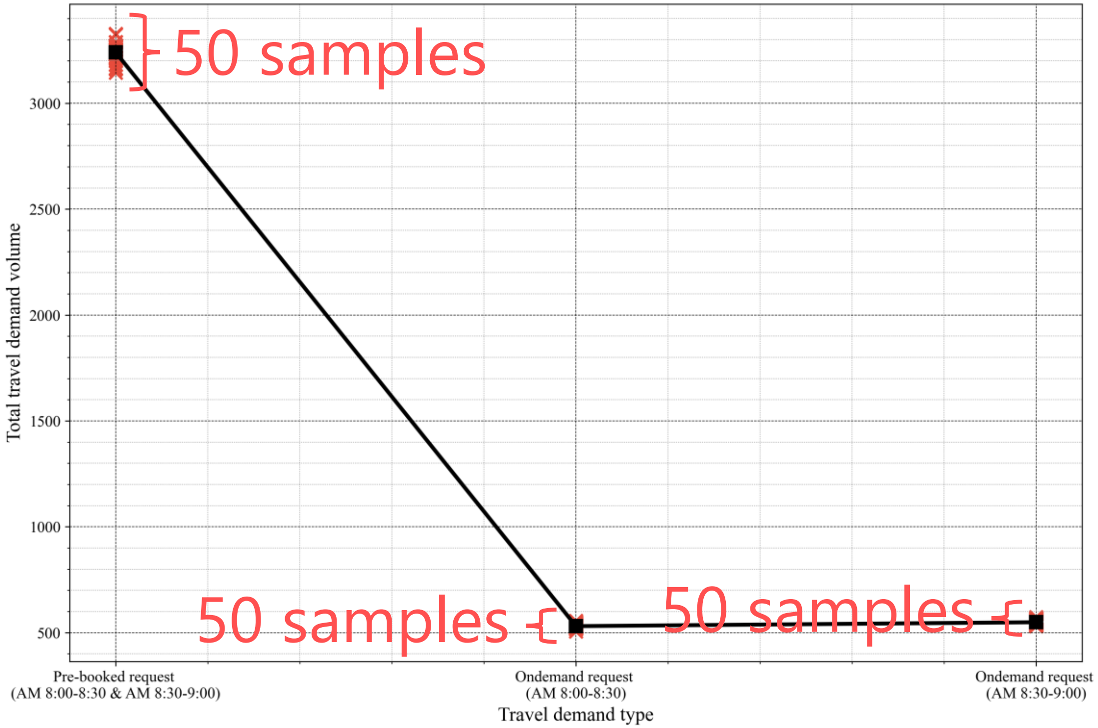
Input Travelers' demand scenarios
(example: reserved rate $p = 0.25$)

Numerical Experiments: Settings

- Travelers' demand was aggregated with a 30 min departure time aggregation width.
- Travelers' demand scenarios was sampled from multivariate uniform distributions.
- We considered 50 samples for pre-booked requests, 50 samples for on-demand requests (AM 8:00-8:30), and 50 samples for on-demand requests (AM 8:30-9:00). The total number of scenarios was 50^3=125,000.



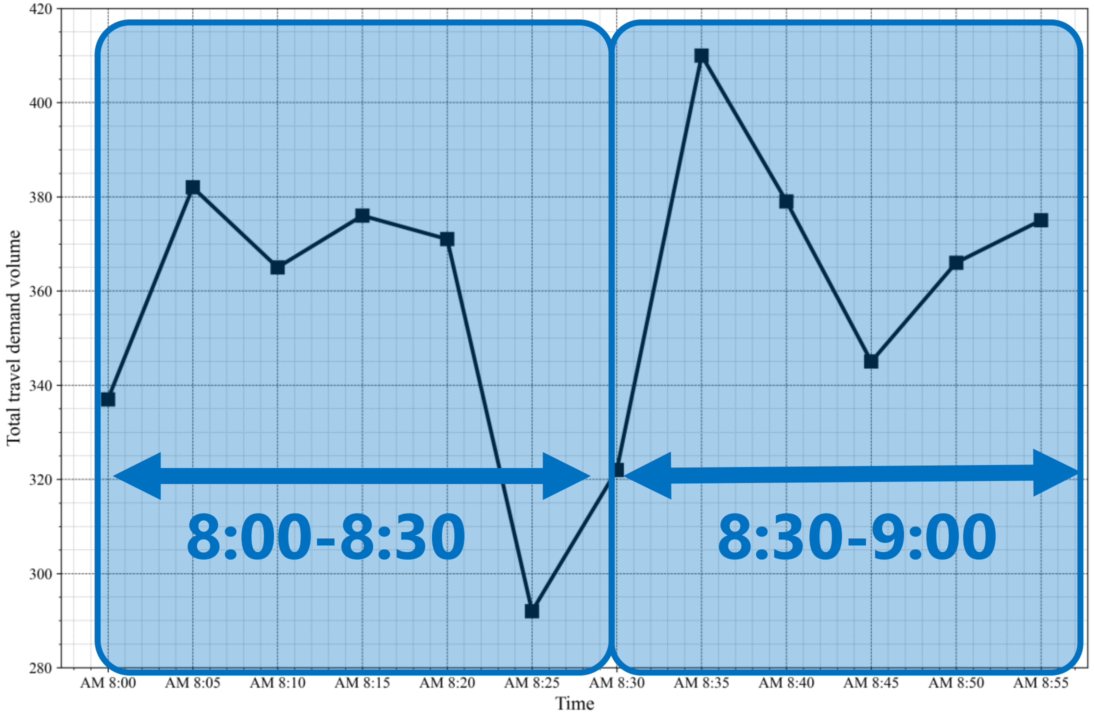
Time-dependent Travelers' demand
(generated from the NYC taxi data)



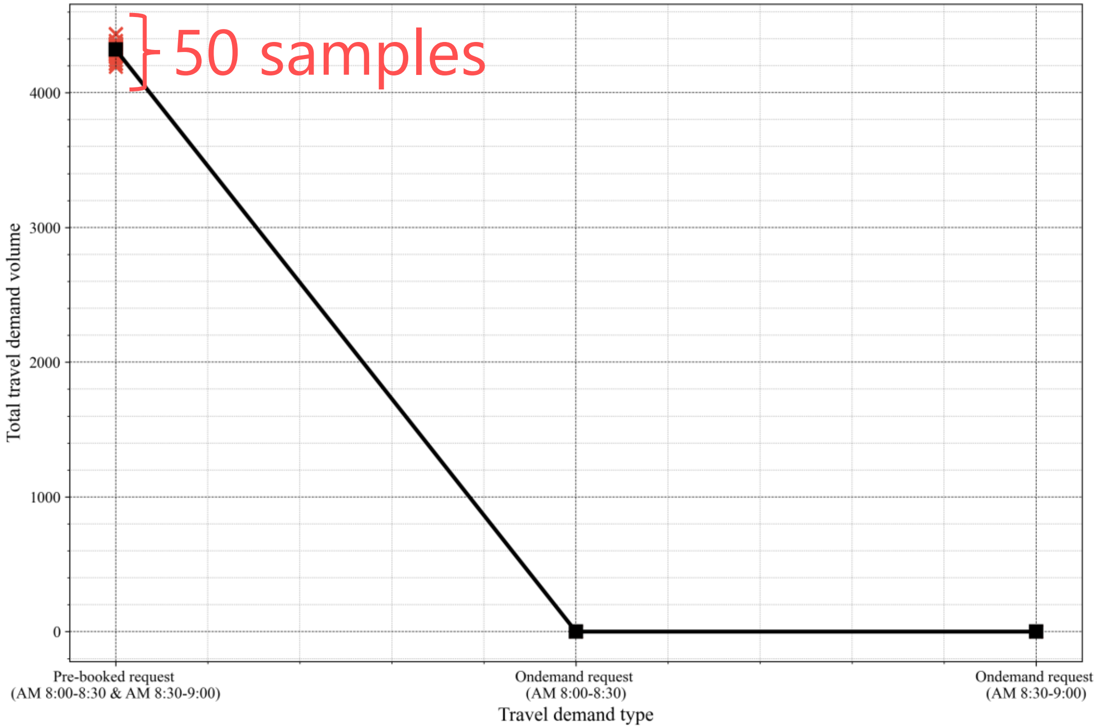
Input Travelers' demand scenarios
(example: reserved rate $p = 0.75$)

Numerical Experiments: Settings

- Travelers' demand was aggregated with a 30 min departure time aggregation width.
- Travelers' demand scenarios was sampled from multivariate uniform distributions.
- We considered 50 samples for pre-booked requests, 50 samples for on-demand requests (AM 8:00-8:30), and 50 samples for on-demand requests (AM 8:30-9:00). The total number of scenarios was $50^3 = 125{,}000$.

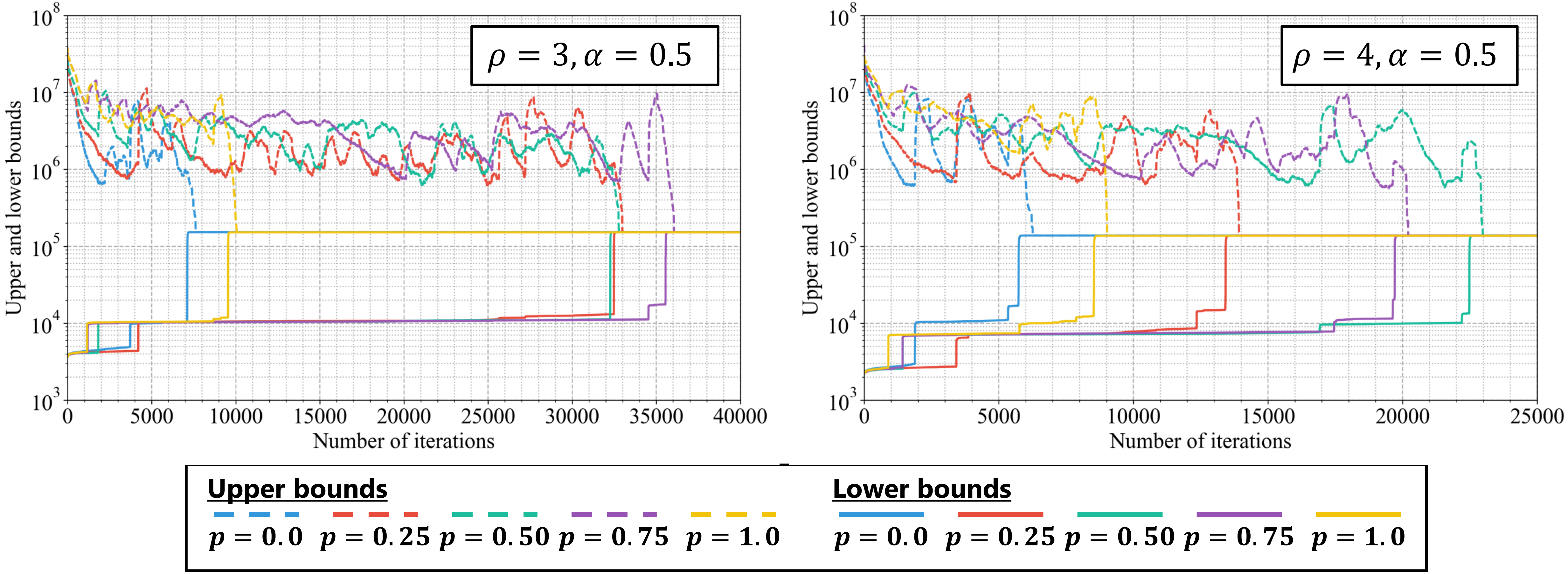


Time-dependent Travelers' demand
(generated from the NYC taxi data)



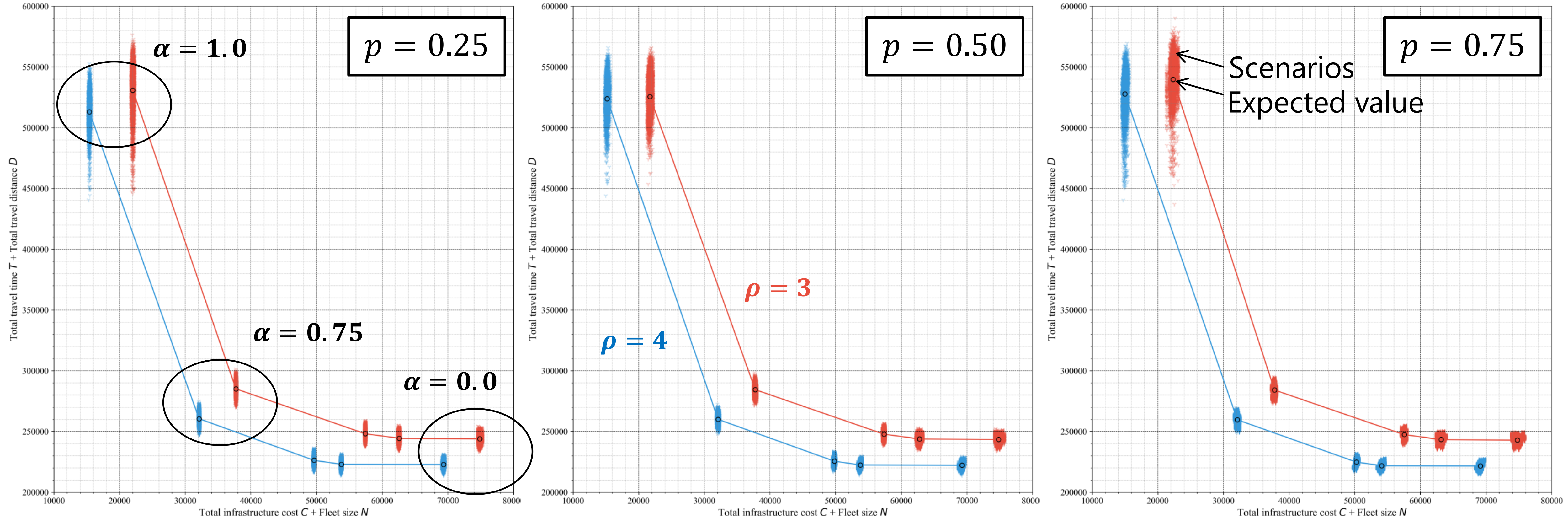
Input Travelers' demand scenarios
(example: reserved rate $p = 1.0$)

Numerical Experiments: Convergence



- The optimal solution can be obtained with a sufficient iterations.
- Note that to obtain the optimal solution in some cases (e.g., $p = 0.75$), it may take a few days, although the solutions in the cases of $p = 0.0$ and 1.0 converge within 24 hours.

Numerical Experiments: Pareto solutions

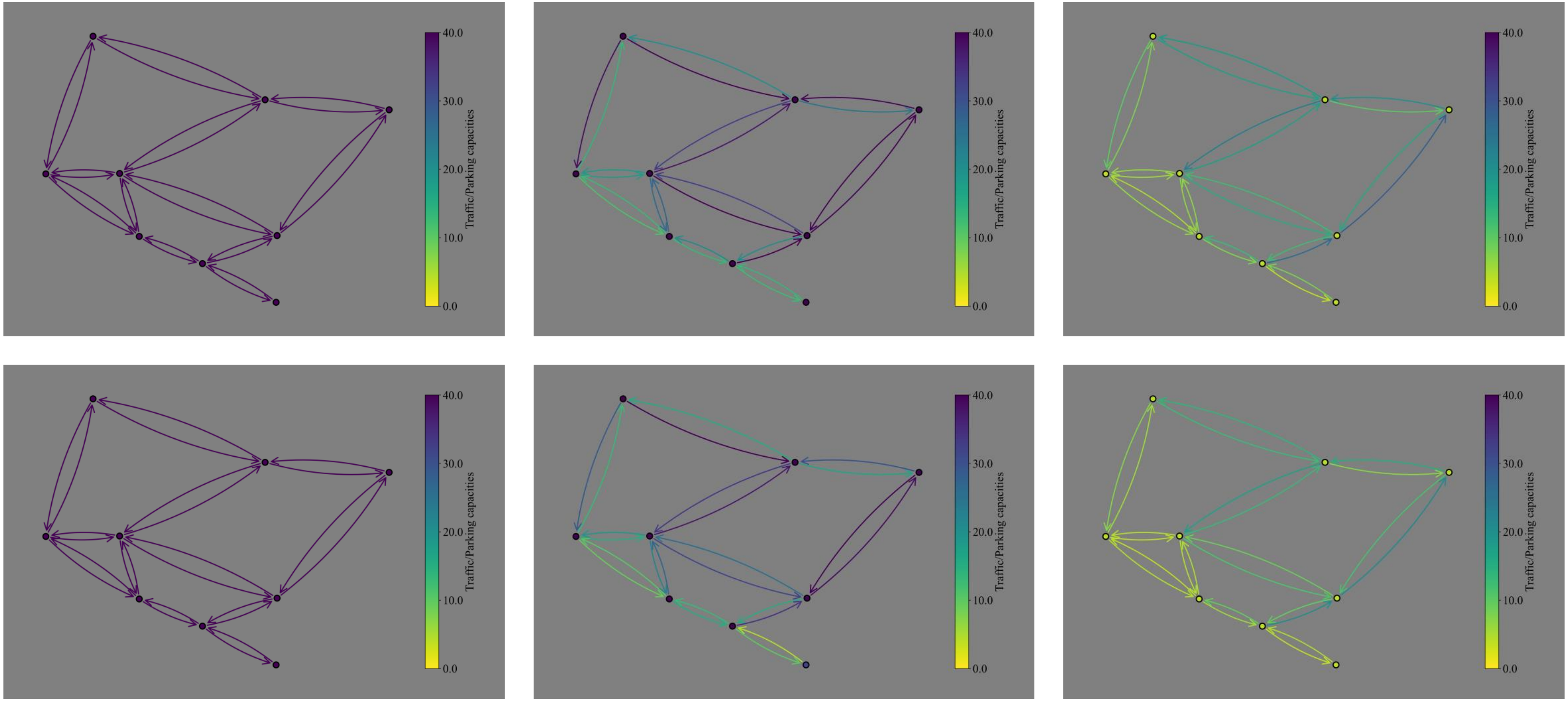


- By comparing $\rho = 3$ to $\rho = 4$, the Pareto-improvement by ridesharing, which is theoretically guaranteed by Theorem 1, was evident.
- In the cases of priority on strategic costs ($\alpha = 1.0$), investments in infrastructures and SAV fleets are reduced, resulting in a greater variance in operating costs.

Numerical Experiments: Infrastructure pattern

$\rho = 3$

$\rho = 4$



$\alpha = 0.0$

$\alpha = 0.5$

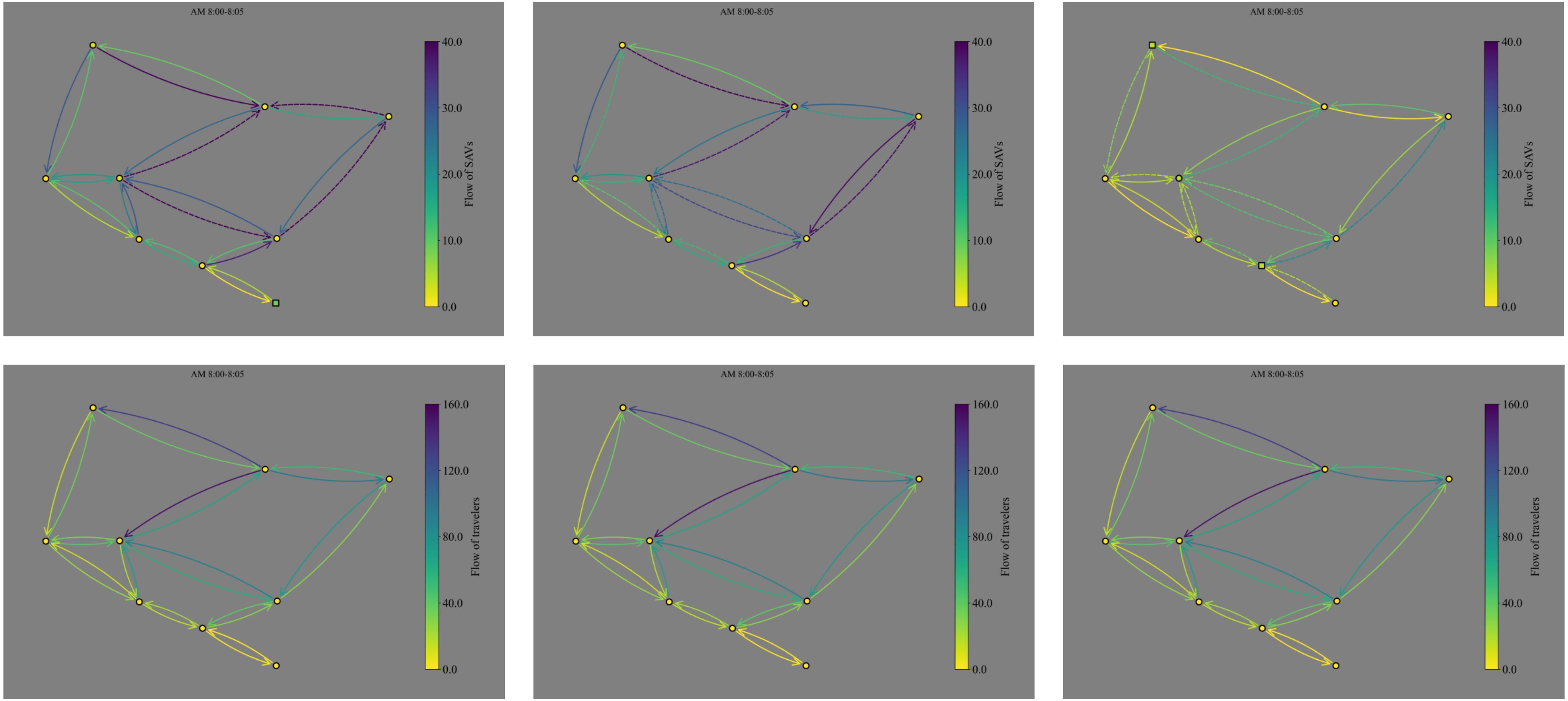
$\alpha = 1.0$

Priority on operation costs ⟺ Priority on strategic costs

Numerical Experiments: Flow pattern

SAV

Traveler



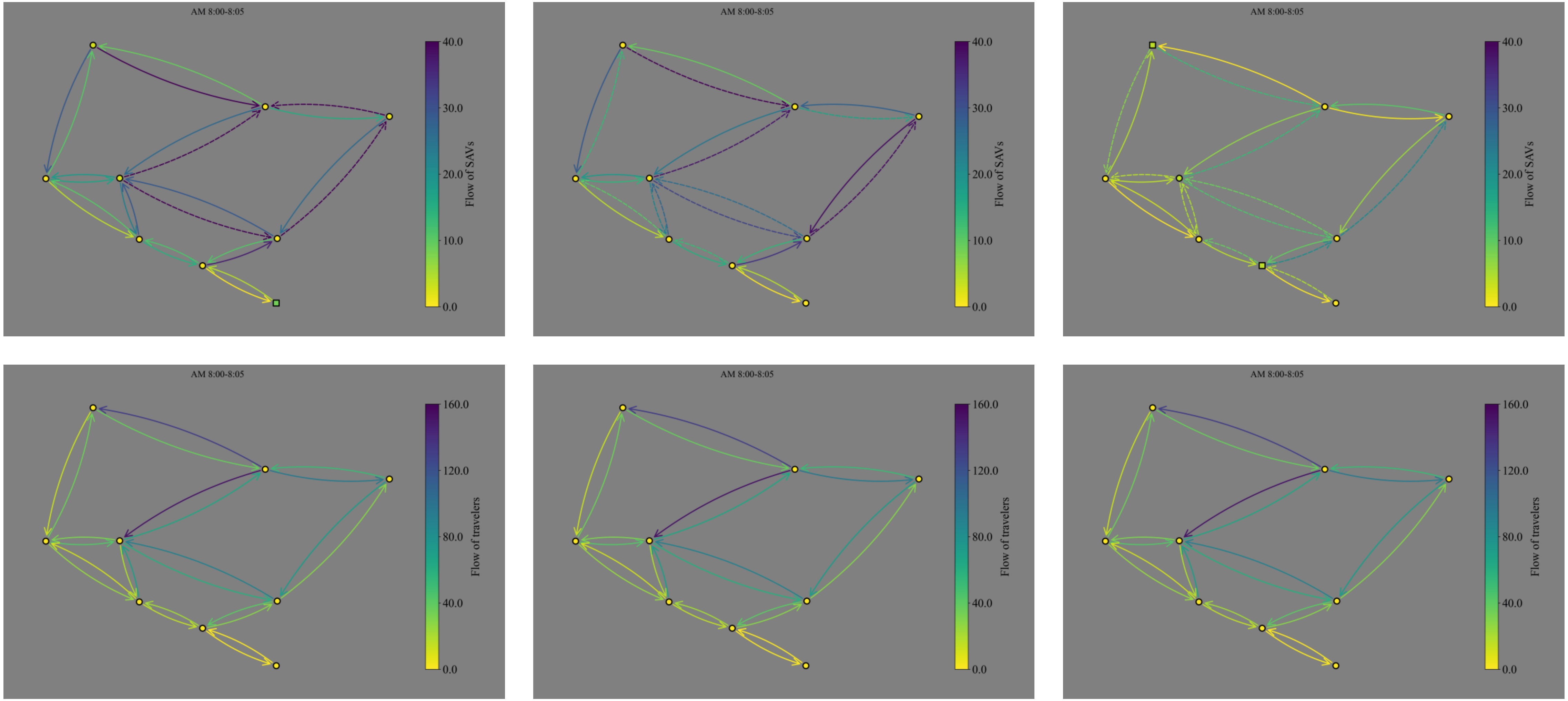
$\alpha = 0.0$ | $\alpha = 0.5$ | $\alpha = 1.0$

Priority on operation costs ⟺ Priority on strategic costs

Numerical Experiments: Flow pattern

SAV

Traveler



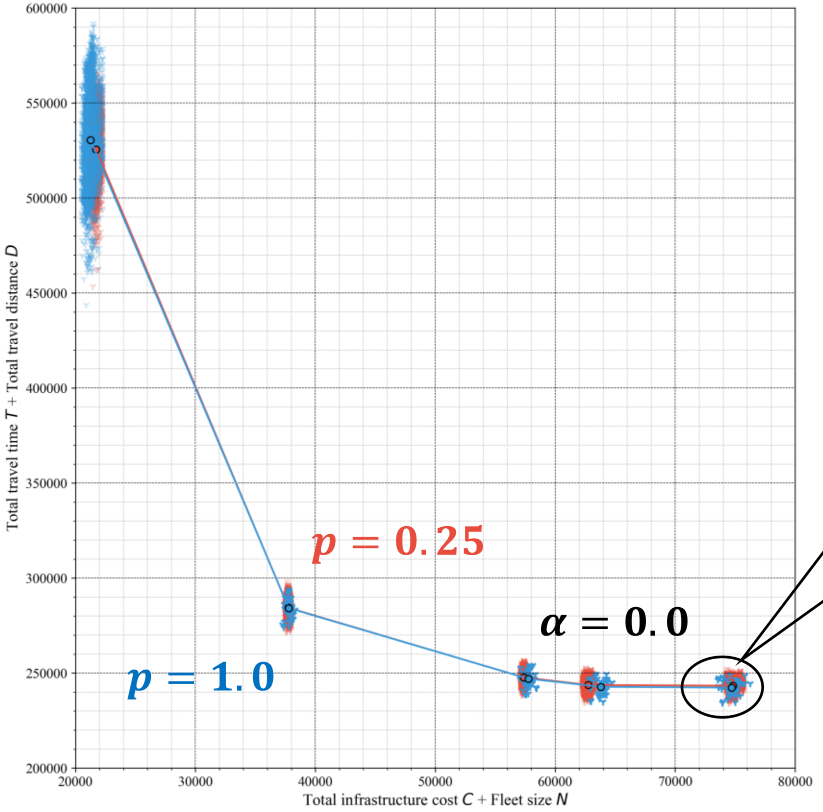
$\alpha = 0.0$

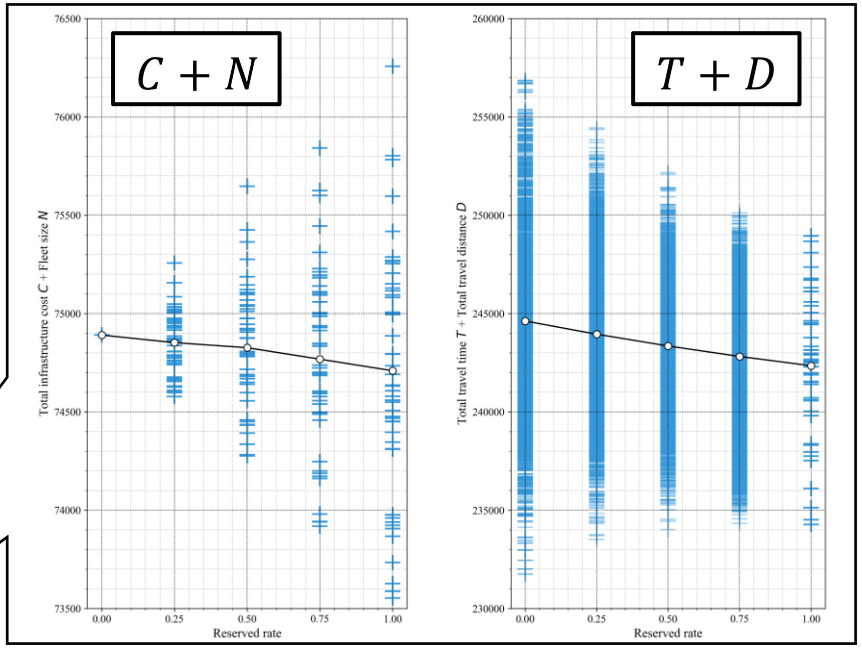
$\alpha = 0.5$

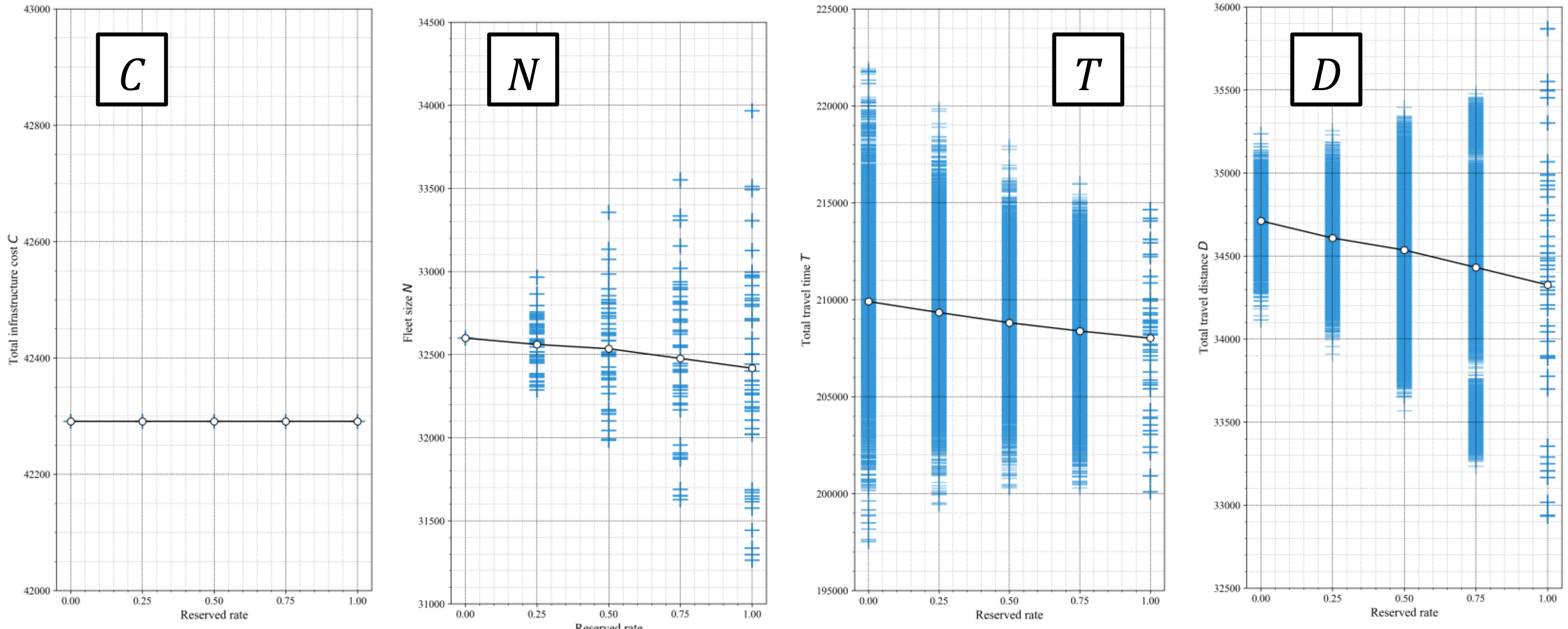
$\alpha = 1.0$

Priority on operation costs ⟺ Priority on strategic costs

Numerical Experiments: Pareto solutions

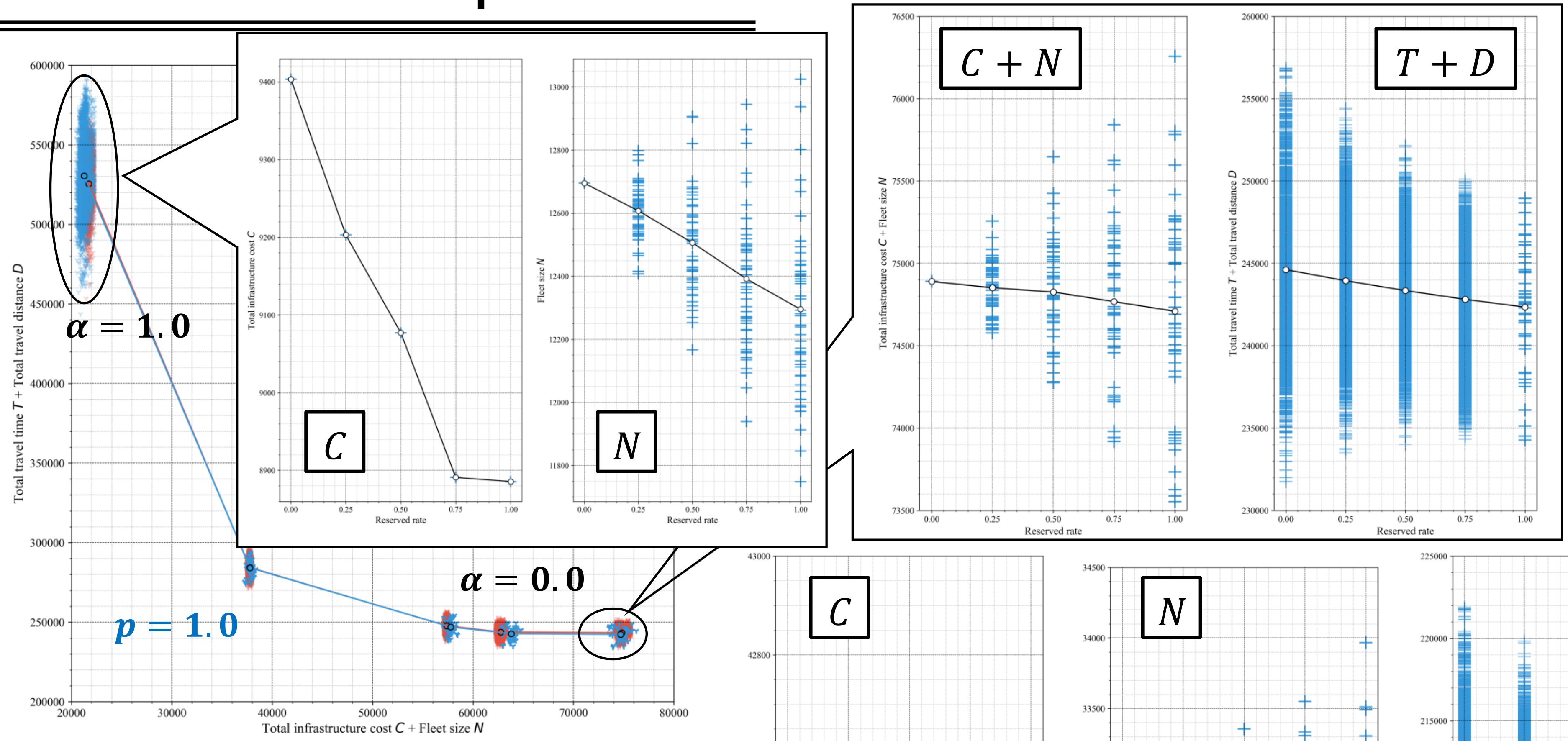


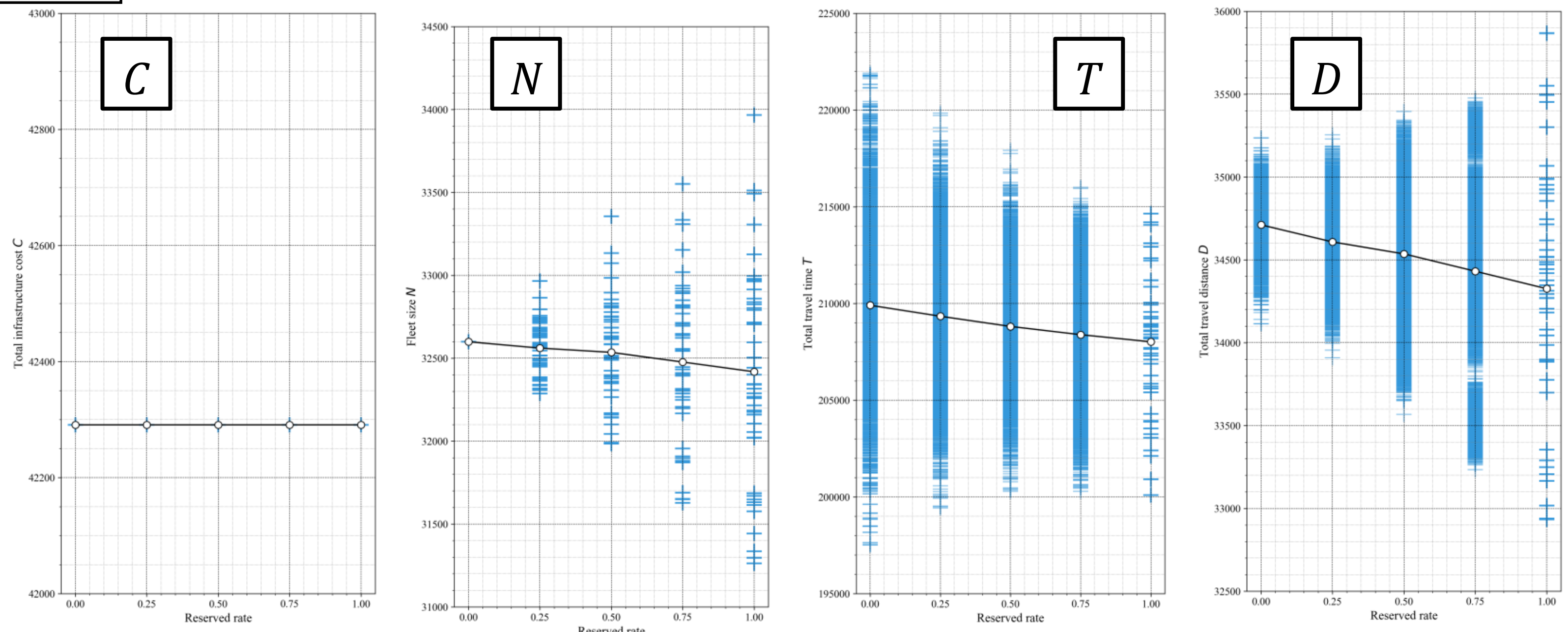




- A slight Pareto improvement by pre-booking options, theoretically guaranteed by Theorem 2, is suggested.

Numerical Experiments: Pareto solutions

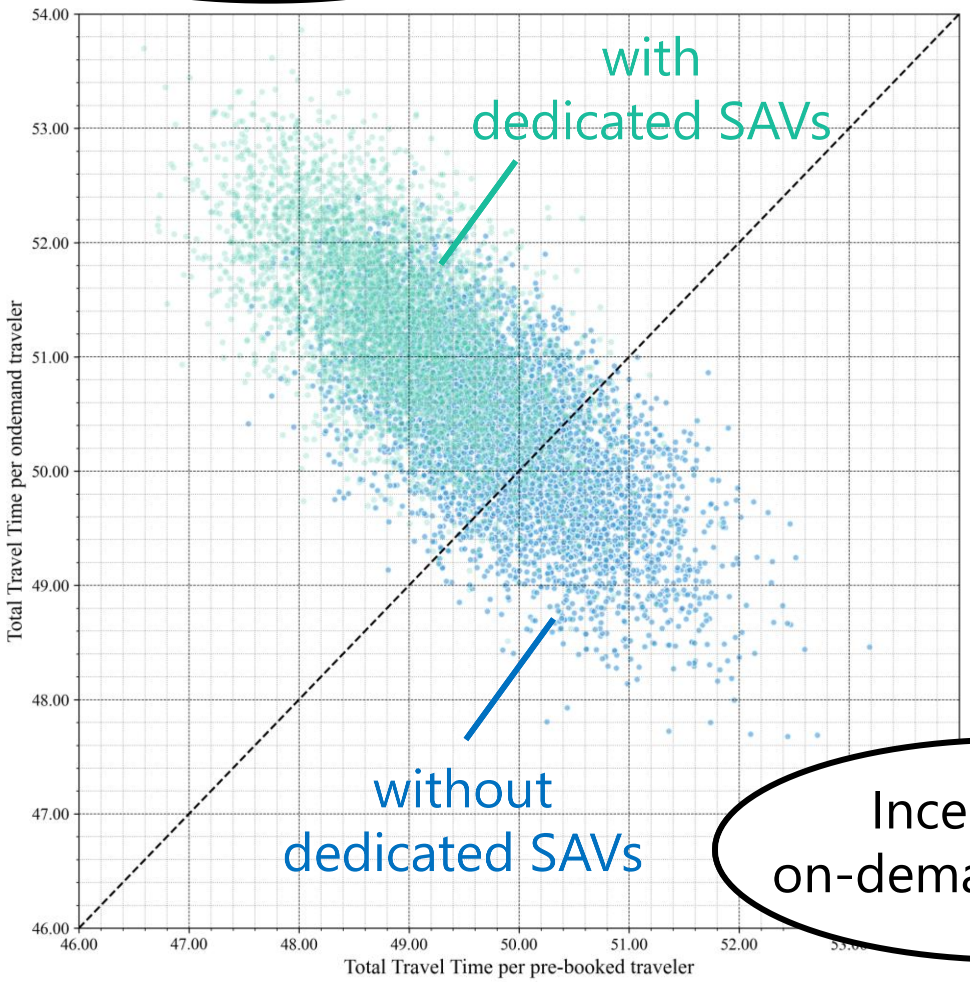




- A slight Pareto improvement by pre-booking options, theoretically guaranteed by Theorem 2, is suggested.

Numerical Experiments: Pre-booking incentives

Incentive for pre-booked requests



Incentive for on-demand requests

How to realize pre-booking SAV system?

- A system design that forces travelers to make reservations will lead to a decrease in their utility.
- To facilitate travelers to pre-book their trips, we introduce dedicated SAVs which provide only pre-booked travelers with pick up and drop off services.
- In the SAV system with dedicated SAVs, average travel time of pre-booked travelers becomes lower than on-demand counterparts.

⇒ The introduction of dedicated vehicles is a promising incentive strategy to encourage travelers to pre-book their trips.

Summary

Summary & Future works

- This study formulates an SAV system design planning and operations under demand uncertainty as a **multi-stage stochastic linear problem**.
- The linearity provides us with the following advantages:
  - SDDP can yield the optimal solution with **guaranteed convergence**.
  - Applying the weighted sum method, we can obtain **Pareto solutions**.
- Future work focuses on **ML-based SDDP** to solve large-scale problems.
  - ML-based SDDP learns an outer approximation of the value function instead of learning the optimal policy.
  - Leveraging the structure of the value function (convex piecewise linear),
    - The solution is guaranteed to be optimal with sufficient iterations, and
    - The computational efficiency is better than simply learning the optimal policy.

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Appendix: Constraints

* To avoid the complexity of notation, let $t_{ij} = 1$.

* Note that accent marks are omitted because the constraints related to pre-booked and on-demand travelers are similar.

<u>Flow Conservation Constraints</u>

SAV flow conservation

$$\sum_{j \in \mathcal{O}_i} x_{ji}^{t-1} + \delta(2,t) x_{0i}^{t-1} = \sum_{j \in \mathcal{I}_i} x_{ij}^{t} + \delta(T,t) x_{i0}^{t}$$

Traveler flow conservation

$$\sum_{j \in \mathcal{O}_i} y_{s,ji}^{k,t-1} + \delta(r,i)\delta(k,t) A_{rs}^{k,t} = \sum_{j \in \mathcal{I}_i} y_{s,ij}^{k,t} + \delta(s,i) y_{s,i0}^{k,t}$$

<u>Demand Constraints</u>

Cumulative departures

$$A_{rs}^{k,t} = A_{rs}^{k,t-1} + \delta^{k,t} \xi_{rs}^{k}$$

Cumulative arrivals

$$D_{s}^{k,t} = D_{s}^{k,t-1} + y_{s,s0}^{k,t}$$

Demand attraction constraints

$$D_{s}^{k,t} = \sum_{r \in \mathcal{R}} A_{rs}^{k,t}$$

<u>Capacity Constraints</u>

Link and node capacity constraints

$$x_{ij}^{t} \leq \mu_{ij}^{t}$$

Vehicle capacity constraints

$$\sum_{k,s} y_{s,ij}^{k,t} \leq \rho x_{ij}^{t}$$